Quantum quenches in quantum field theory: Sine Gordon and the form factor approach

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J. Stat. Mech. (2017) 103106, with Dirk Schuricht
Soon to appear in JSTAT

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Sine Gordon vs. Sheer brute force

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Ingredients of quenches

Initial state $|\Psi_0\rangle$

$|\Psi_t\rangle = e^{-iHt}|\Psi_0\rangle$

Eigenstate basis + overlaps $\langle n|\Psi_0\rangle$

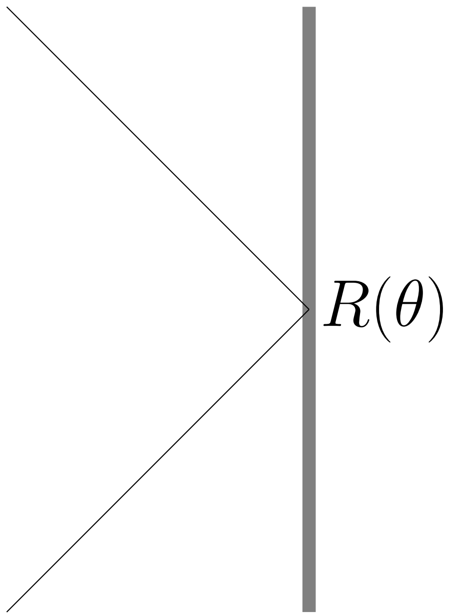
Form factors of local operators $\langle n|\Phi(x)|m\rangle$

Compute correlators...

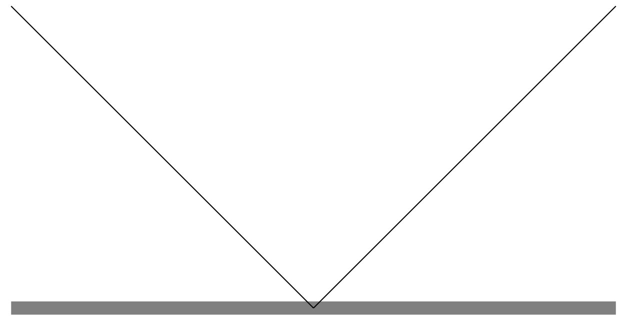
$$\frac{\langle \Psi_t|\Phi_1(x_1)\Phi_2(x_2)...|\Psi_t\rangle}{\langle \Psi_0|\Psi_0\rangle}$$

Initial states

Simplest initial states one can imagine, correspond to nice boundary conditions



Initial states



$K_{\mathrm{reg}}(\theta)R(\mathrm{i}\pi/2-\theta)$

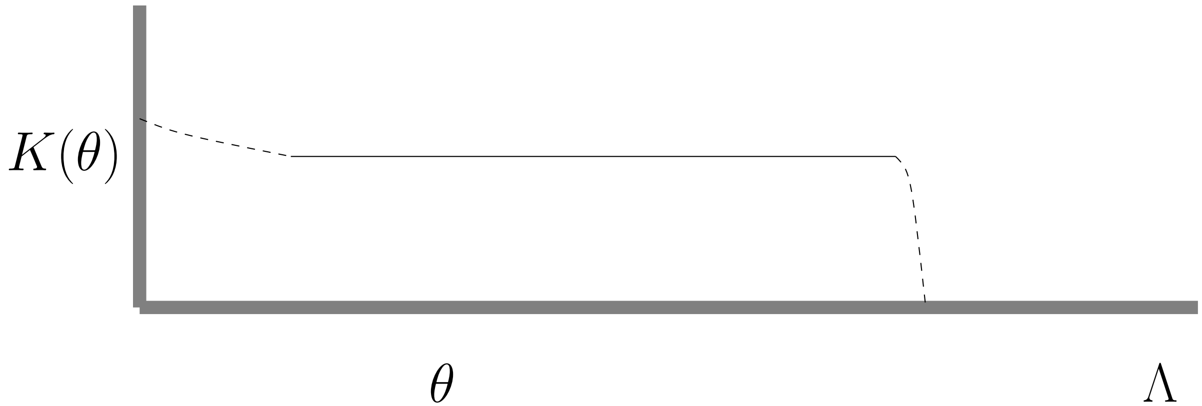
First problem relative to lattice quenches: regularization needed

$$|\Psi_0\rangle = \exp\left(\int K(\theta)A^\dagger(-\theta)A^\dagger(\theta)\right)|0\rangle$$

"Integrable initial states" excite states from all regions of Hilbert space

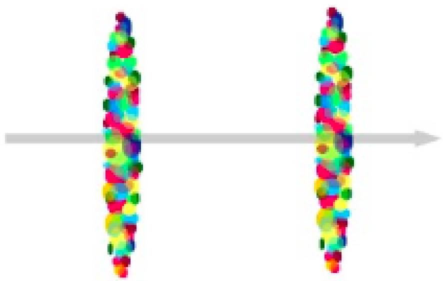
Why QFT quenches then?

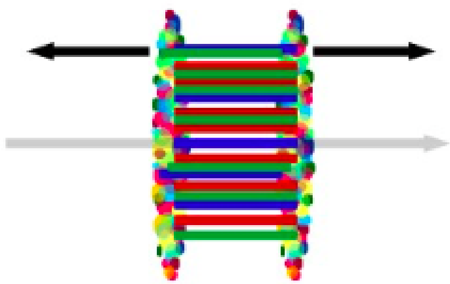
When is a QFT quench justified?



Usual region of validity, "small quenches" where dangerous high-energy excited states are suppressed

Some people care about non equilibrium QFT and have no interest in spin chains





"Integrable initial states"

Do they mean anything?

Probably not, but are probably good enough

Evidence:

Free boson/fermion mass quench $m_0 \to m$, solvable, integrable

Infinite pre-quench mass quenches (Bad QFT)

Some particular mass-quench protocols in sinh Gordon and sine Gordon at least well approximated by integrable initial state
Sotiriadis, Takacs, Mussardo, Horvath

Form factor axioms

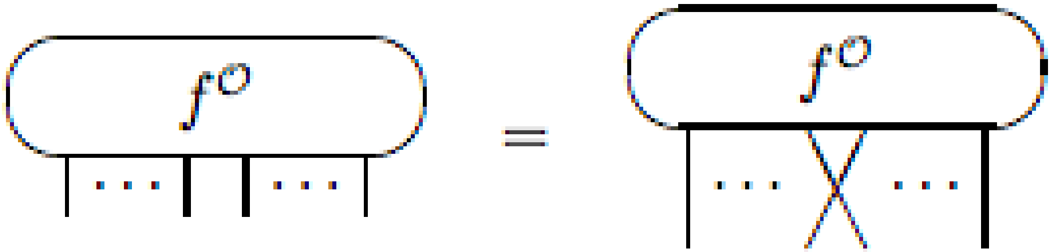
Relativistic QFT states parametrized by particle rapidities,

$$E = m\cosh\theta, \quad p = m\sinh\theta$$

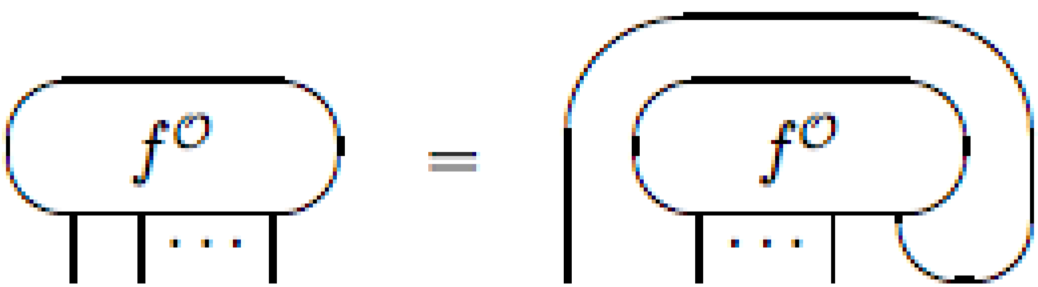
We now seek to compute form factors

$$F(\theta_1, \ldots, \theta_n) = \langle 0|\mathcal{O}(x)|\theta_1, \ldots, \theta_n\rangle$$

Form factor axioms

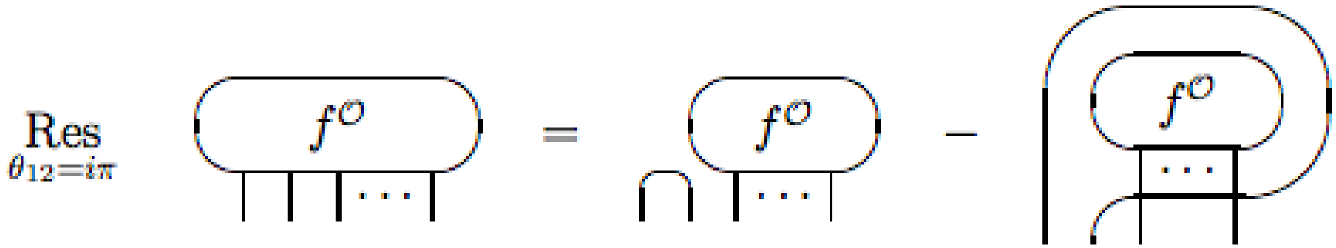


Scattering axiom



Periodicity/crossing

Annihilation pole axiom and semi-locality



Recursive equations of the form $f_n \to f_{n-2}$

until f_2, if particles are local w.r.t. the operator, until f_0 if semilocal.

Curse+blessing of relativistic QFT quench dynamics: poles

$\langle\{\theta'\}|\mathcal{O}|\{\theta\}\rangle$ have poles at real rapidities

Form factor approach to time evolution in quenches

$$\langle\Psi_t|\mathcal{O}|\Psi_t\rangle = \sum_{M,N}\int_0^\infty \frac{d\theta'_1\dots d\theta'_M}{M!(2\pi)^M}\frac{d\theta_1\dots d\theta_N}{N!(2\pi)^N}$$

$$\times\prod_m (K(\theta_m))^*\prod_n K(\theta_n)$$

$$\times e^{2mit\sum_m\cosh\theta'_m - 2mit\sum_n\cosh\theta_n}$$

$$\times\langle\{\theta'_m\},\{-\theta'_m\}|\mathcal{O}|\{-\theta_n\},\{\theta_n\}\rangle$$

Form factor approach to time evolution in quenches

Simple set up, yet not a lot more

Ising field theory
D. Schuricht and F. H. L. Essler (2012)

repulsive sine Gordon
B. Bertini, D. Schuricht and F. H. L. Essler (2014)

$SU(N) \times SU(N)$ Principal chiral sigma model, $N \to \infty$
A.C.C. (2016)

Repulsive regime of sine Gordon

Bertini, Schuricht, Essler (2014)

$$H = \frac{1}{16\pi}\int dx\,\left[\left(\partial_x\Phi\right)^2 + \left(\partial_t\Phi\right)^2\right] - \lambda\cos\left(\beta\Phi\right)$$

$$\beta^2 > 1/2$$

Spectrum consists of solitons and antisolitons

$$|\theta_1,\theta_2,\ldots\rangle_{a,b,\ldots}$$

These are semiclassically associated with kinks of the bosonic field, Φ.

Solitons are semilocal with respect to, for example, vertex operators $e^{\mathrm{i}\alpha\Phi}$.

Annihilation poles vs late time evolution

$$\langle\Psi_t|\mathcal{O}|\Psi_t\rangle = \sum_{M,N}\int_0^\infty \frac{d\theta_1'\ldots d\theta_M'}{M!(2\pi)^M}\frac{d\theta_1\ldots d\theta_N}{N!(2\pi)^N}$$

$$\times\prod_m (K(\theta_m))^*\prod_n K(\theta_n)$$

$$\times e^{2mit\sum_m \cosh\theta_m' - 2mit\sum_n\cosh\theta_n}$$

$$\times\langle\{\theta_m'\},\{-\theta_m'\}|\mathcal{O}|\{-\theta_n\},\{\theta_n\}\rangle$$

Late times: perform integrals through stationary phase, leads to $1/t^\gamma$ terms

Only annihilation poles can cut through these s.p. integrals, produce leading late-time terms

Leading time given by most divergent region of form factors!

First leading term for vertex operator: 1 pair + 1 pair

$$\sim \int \frac{d\theta' d\theta}{(2\pi)^2} \left(K(\theta')\right)^* K(\theta) e^{2mit(\cosh\theta' - \cosh\theta)}$$

$$\times \langle \theta', -\theta' | e^{i\beta\Phi/2} | -\theta, \theta \rangle$$

Two annihilation poles vs two rapidity integrals.

Need to regularize with finite volume and carefully massage

$$= -\langle e^{i\beta\Phi/2} \rangle \frac{t}{\tau}, \qquad \frac{1}{\tau} = \frac{2m}{\pi} \int d\theta |K(\theta)|^2 \sinh\theta + \mathcal{O}(K^2)$$

+ S.P. time-decaying stuff

Leading terms

Terms with the most annihilation poles compared to the number of integrals

Only "diagonal" terms with 0, 1, 2,... pairs

Compute term by term:

$$\frac{\langle \Psi_t | e^{\mathrm{i}\beta\Phi/2} | \Psi_t \rangle}{\langle \Psi_0 | \Psi_0 \rangle} = \langle e^{\mathrm{i}\beta\Phi/2} \rangle \left[1 - \frac{t}{\tau} + \frac{1}{2} \left(\frac{t}{\tau} \right)^2 + \ldots \right]$$

Resum!

$$= \langle e^{\mathrm{i}\beta\Phi/2} \rangle e^{-t/\tau}$$

Sine Gordon in the attractive regime

A.C.C. and D. Shuricht (2017)

$$H = \frac{1}{16\pi}\int dx\,\left[(\partial_x\Phi)^2 + (\partial_t\Phi)^2\right] - \lambda\cos(\beta\Phi)$$

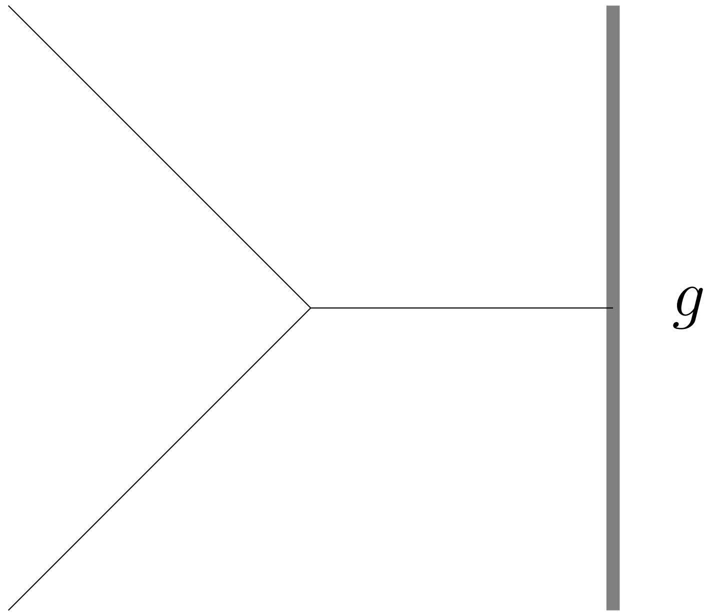
$$\beta^2 < 1/2$$

Spectrum of "breather" bound states, $n-1$ breather species, for $\beta^2 < 1/n$

Topologically trivial (not kinks!)

particles are local with respect to the bosonic field!

Boundary bound states



Initial state with boundary bound states

$$|\Psi_0\rangle = \Big(1 + gA_B^\dagger(0)\Big)\exp\left(\int K(\theta)A^\dagger(-\theta)A^\dagger(\theta)\right)|0\rangle$$

Quench spectroscopy

V. Gritsev, E. Demler, M. Lukin, and A. Polkovnikov (2007)

Define power spectrum

$$P(\omega) = \lim_{t\to\infty} \left| \int_0^T dt e^{\mathrm{i}\omega t} \frac{\langle \Psi_t | e^{\mathrm{i}\alpha\Phi} | \Psi_t \rangle}{\langle \Psi_0 | \Psi_0 \rangle} \right|^2$$

Experimentally accessible

Boundary bound state leading contributions are permanent oscillations $\frac{\langle \Psi_t | e^{\mathrm{i}\alpha\Phi} | \Psi_t \rangle}{\langle \Psi_0 | \Psi_0 \rangle} \sim |g| f_B \cos(m_B t - \delta)$

leads to delta function contributions in the Power spectrum, at breather masses!

Problems for the attractive regime

Breather form factors have one less annihilation pole, compared to soliton form factors.

Breathers do not contribute to the late time dynamics?
Not true as one can see from quench spectroscopy picture

What is the role of boundary bound states on the late-time dynamics?

Leading late-time contributions

The problem is its own solution!

In the presence of boundary bound states, breathers are allowed to have the extra needed annihilation pole

$$\langle\theta'|e^{\mathrm{i}\beta\Phi/2}|\theta\rangle \to \text{ nothing } \quad :($$

$$\langle\theta'|e^{\mathrm{i}\beta\Phi/2}|0_B,\theta\rangle \to \langle 0|e^{\mathrm{i}\beta\Phi/2}|0_B\rangle$$

$$\langle\theta',0_B|e^{\mathrm{i}\beta\Phi/2}|0_B,\theta\rangle \to \langle 0_B|e^{\mathrm{i}\beta\Phi/2}|0_B\rangle$$

Construct the leading late-time contributions **on top** of the boundary bound states!

Resummation of leading late-time terms

Leading terms: 0 pairs +BBS, 1 pair +BBS, 2 pairs+ BBS,...

$$\frac{\langle \Psi_t | e^{\mathrm{i}\alpha\Phi} | \Psi_t \rangle}{\langle \Psi_0 | \Psi_0 \rangle} = \left[\langle e^{\mathrm{i}\beta\Phi/2} \rangle + \frac{|g|^2}{4} f_B B(\mathrm{i}\pi, 0) \right] e^{-t/\tau} + |g| f_B \cos(\Omega t - \delta) e^{-t/\tau_2}$$

Oscillatory terms aquire exponential damping too!, also oscillation frequency receives a shift of "order K^2"

Softened and less accurate quench spectroscopy

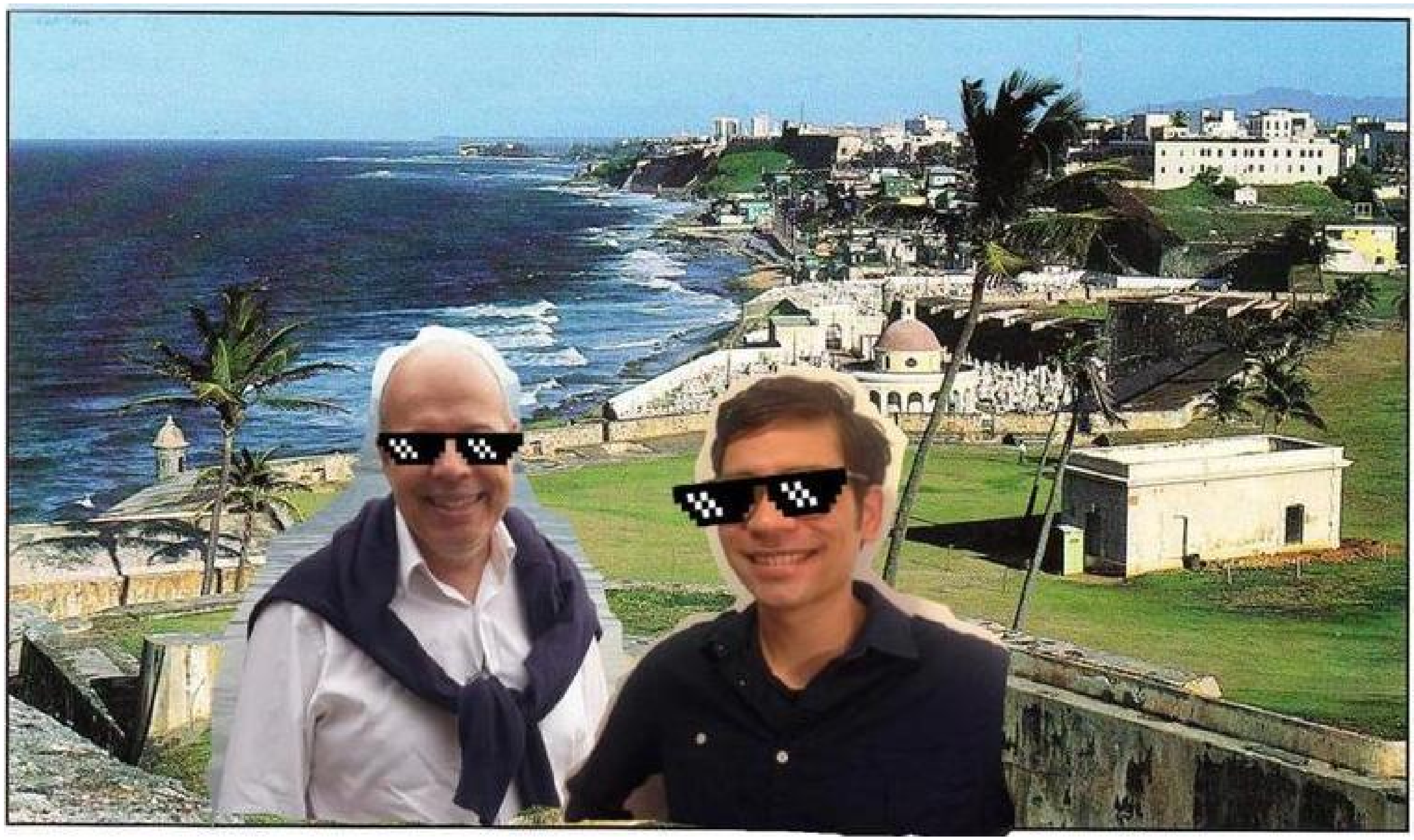
Compute power spectrum

$$P(\omega) = \lim_{t\to\infty} \left| \int_0^T dt e^{\mathrm{i}\omega t} \frac{\langle \Psi_t | e^{\mathrm{i}\alpha\Phi} | \Psi_t \rangle}{\langle \Psi_0 | \Psi_0 \rangle} \right|^2$$

Exponential damping $\rightarrow$ sharp delta-function peaks become soft Lorentzian peaks

Small shift in oscillation frequency $\rightarrow$ The location of the peak is not exactly on the breather masses, intrinsic uncertainty

Happy Birthday!



Making up for lost time!