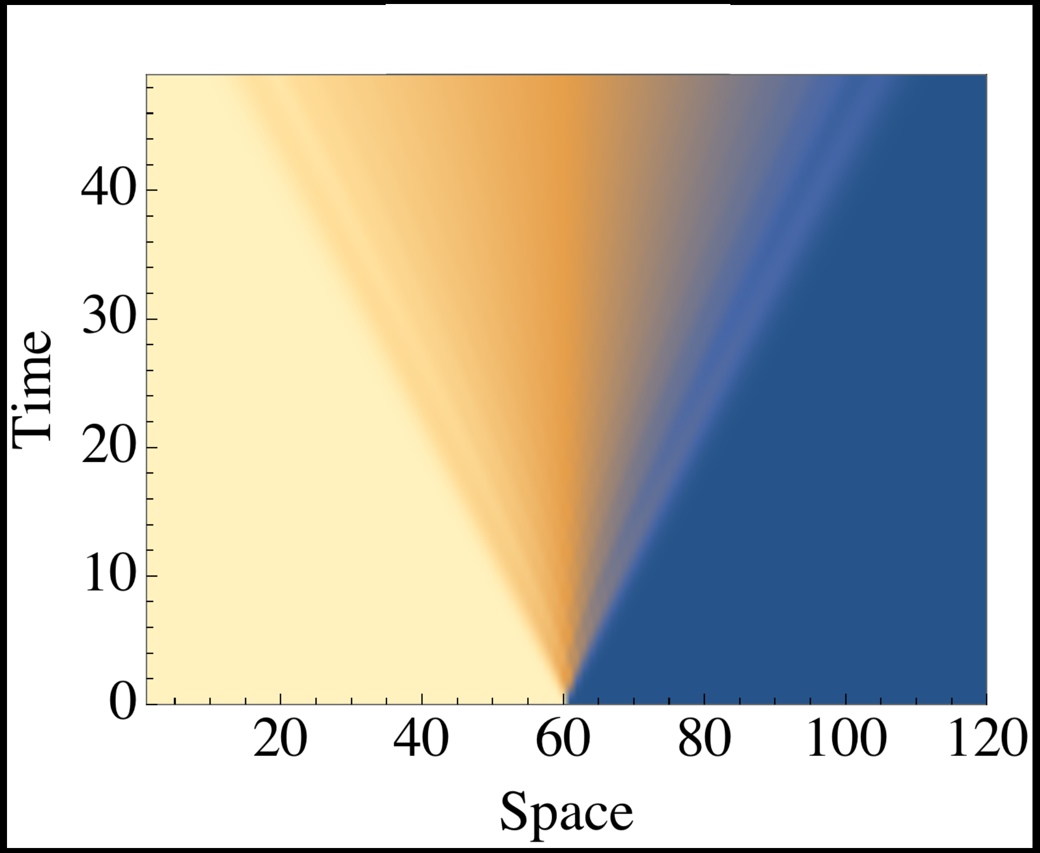
Hydrodynamics and density-density correlations of the one-dimensional Bose gas



Jacopo De Nardis

In collaboration with:

Enej Ilievski UvA
Maurizio Fagotti ENS
Andrea De Luca Oxford
Miłosz Panfil Warsaw
Mario Collura Oxford
Lorenzo Piroli SISSA

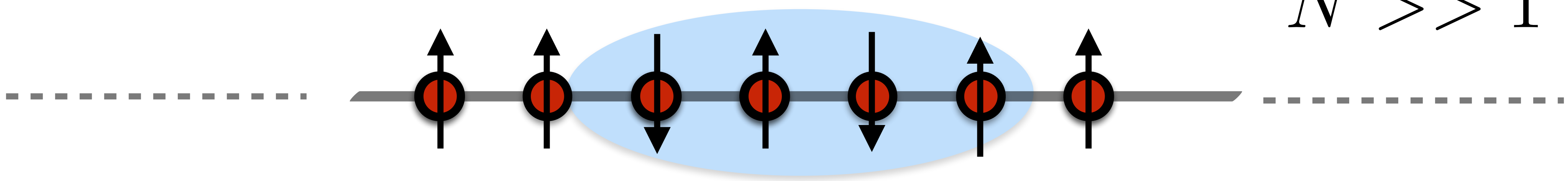
and more

ENS
ÉCOLE NORMALE SUPÉRIEURE

Correlation functions in integrable quantum systems

$$\langle \sigma_1(t)\sigma_n(0)\rangle$$

$$N >> 1$$



Correlation functions in integrable quantum systems

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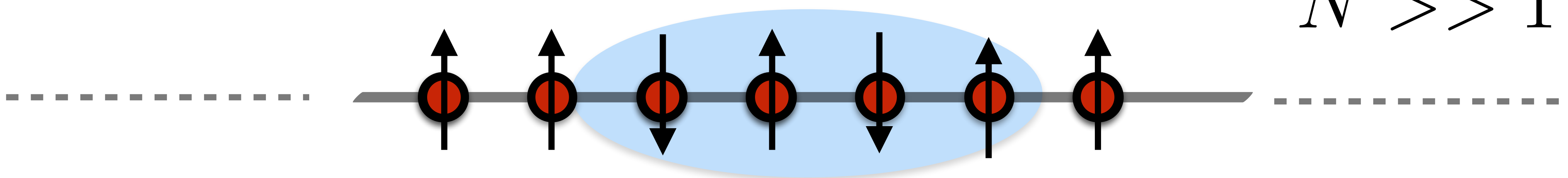
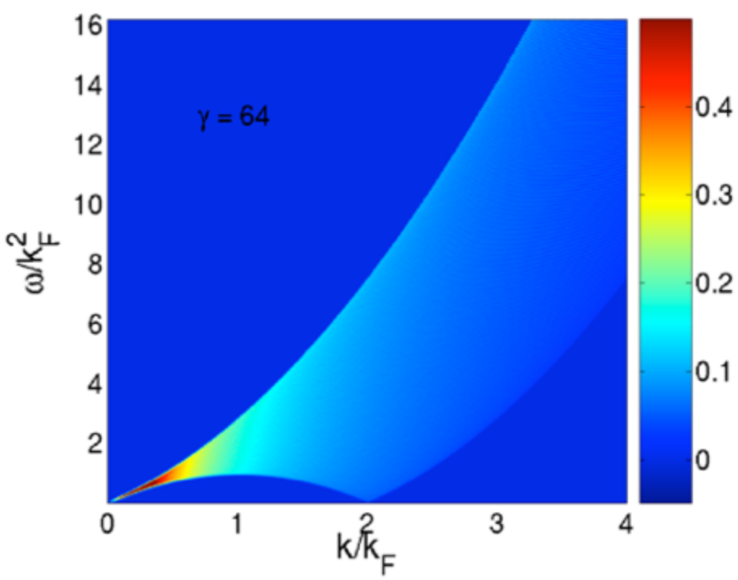
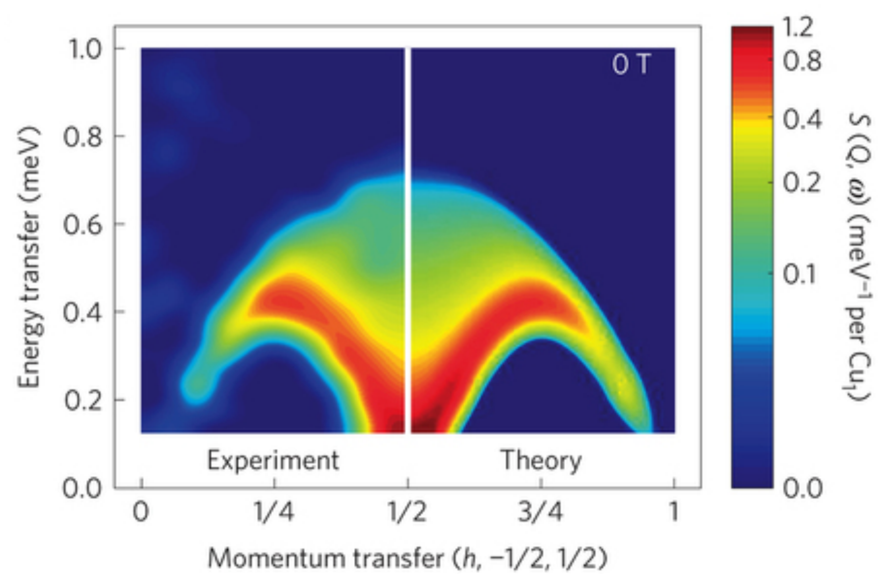
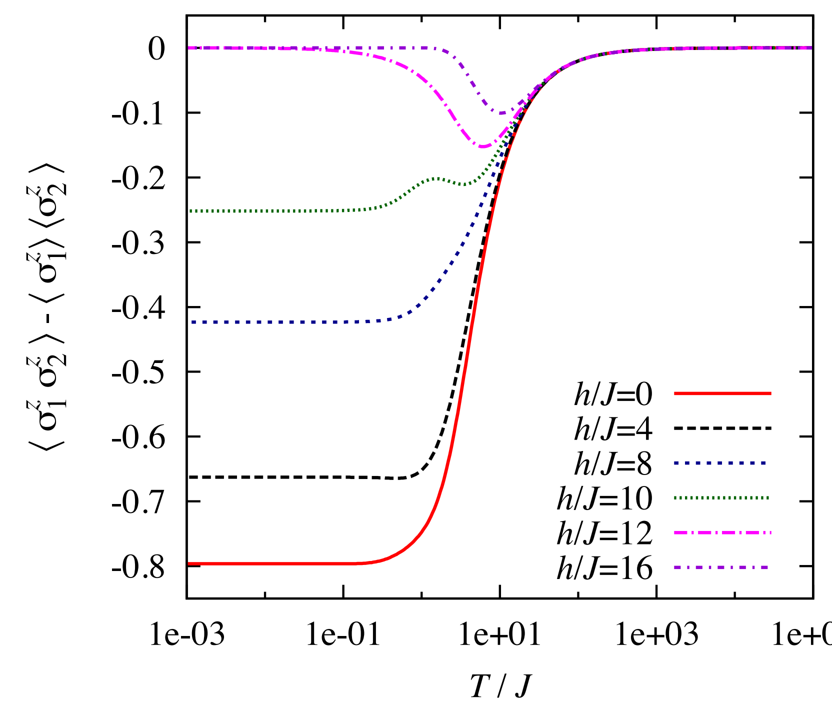




Caux and Calabrese, PRA (2008)



M. Mourigal et al, Nature Physics (2013)

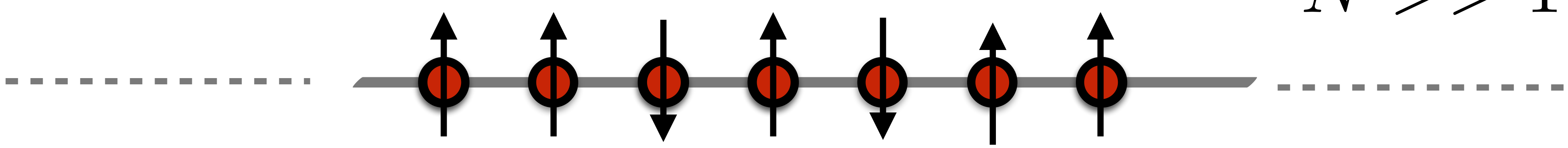


Trippe, Göhmann, Klümper EPJ (2010)

Non-equilibrium dynamics from homogenous states

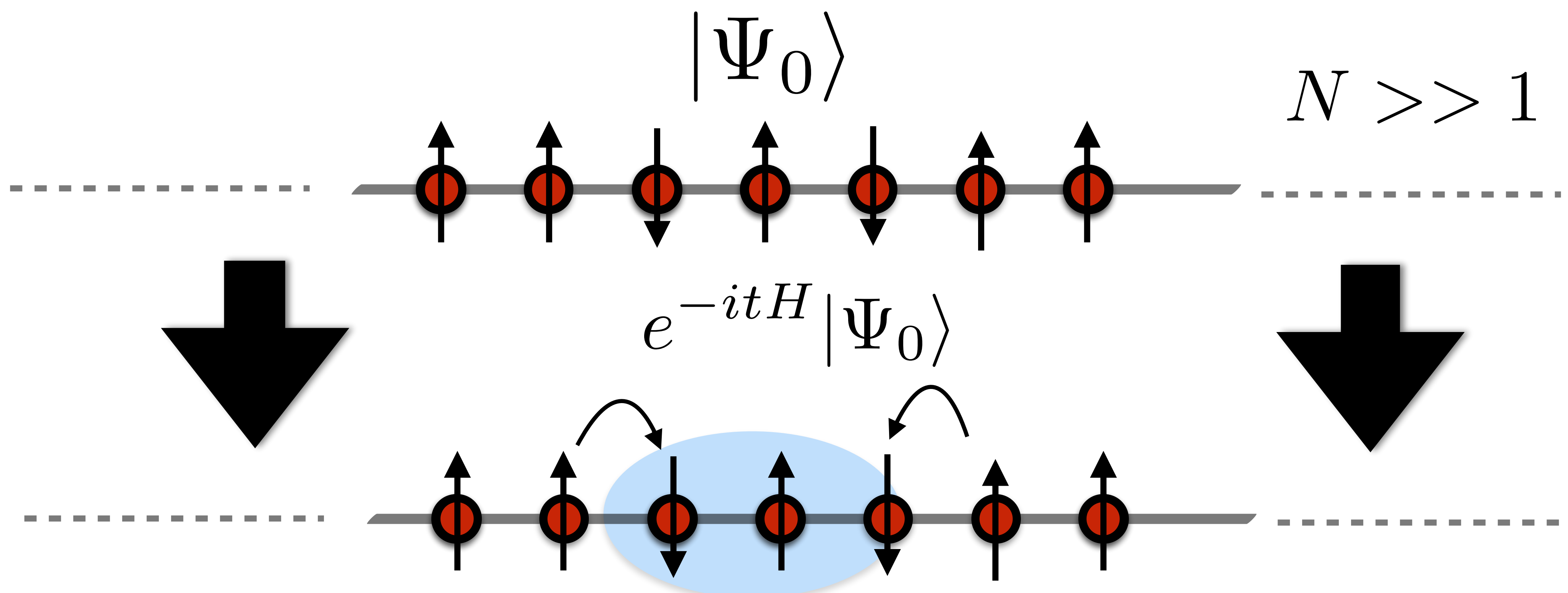
$$|\Psi_0\rangle$$

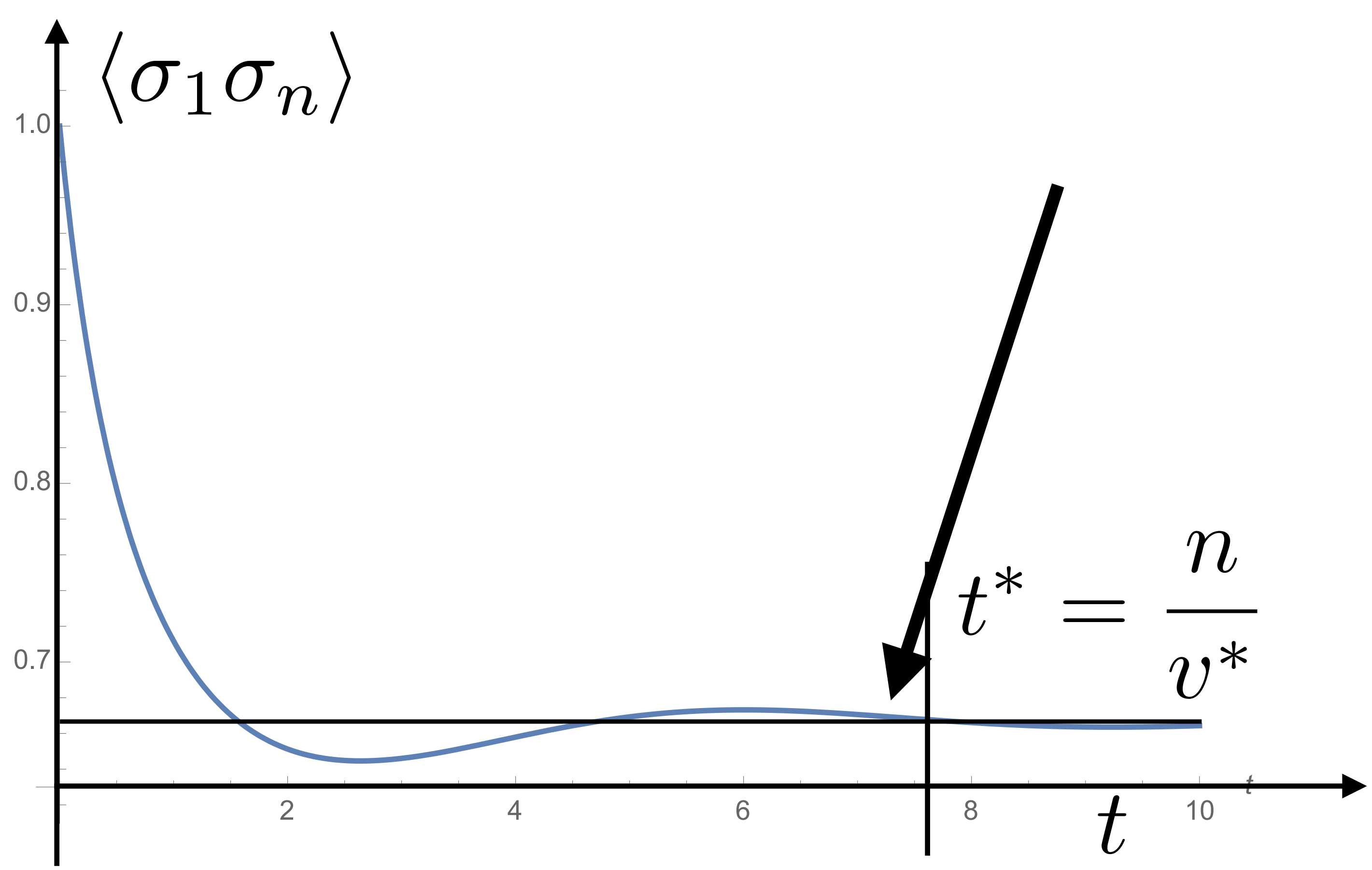
$$N >> 1$$



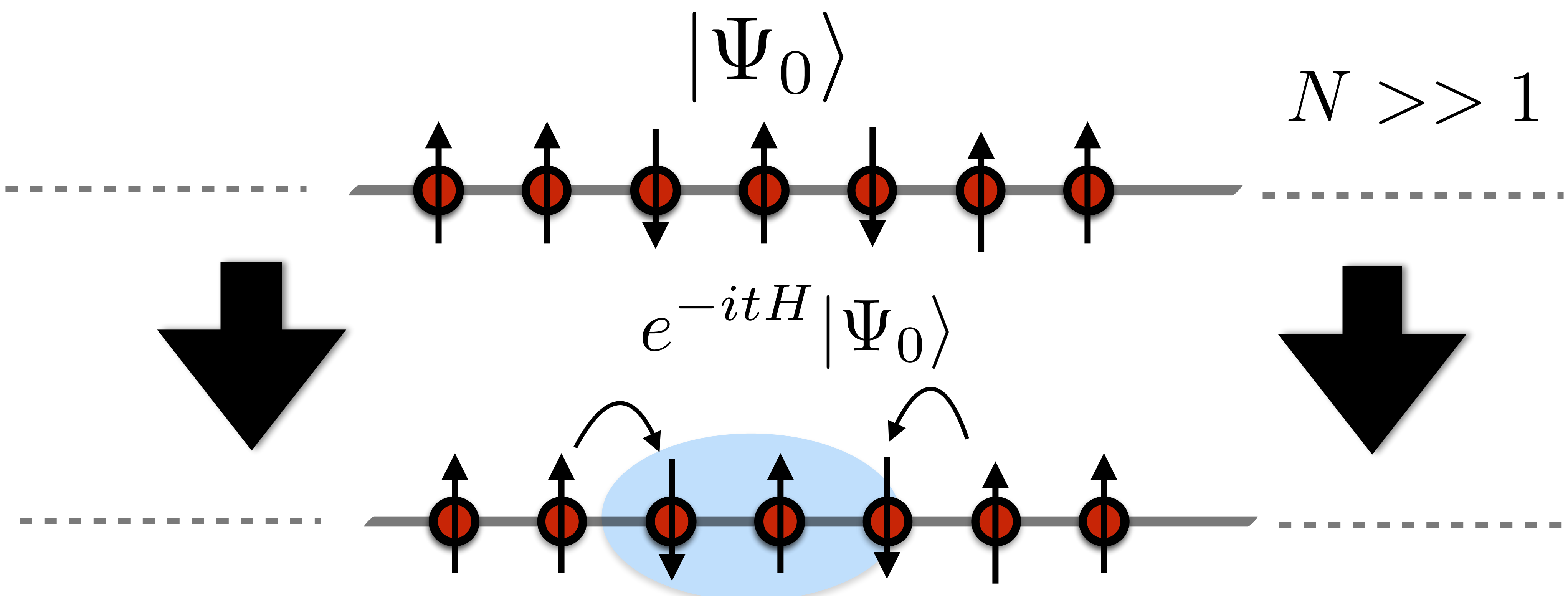
$$e^{-itH}|\Psi_0\rangle$$

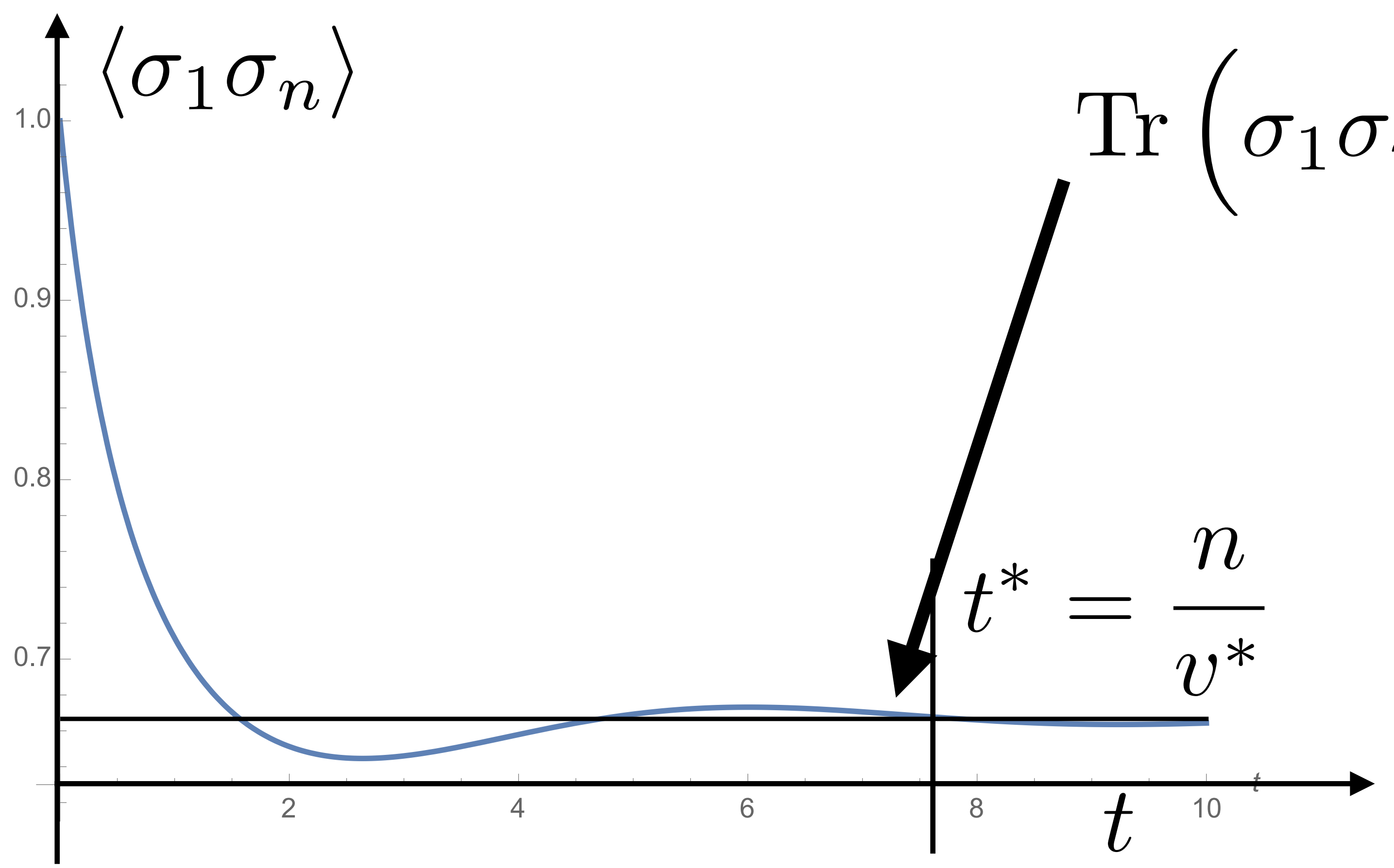
Non-equilibrium dynamics from homogenous states





Non-equilibrium dynamics from homogenous states



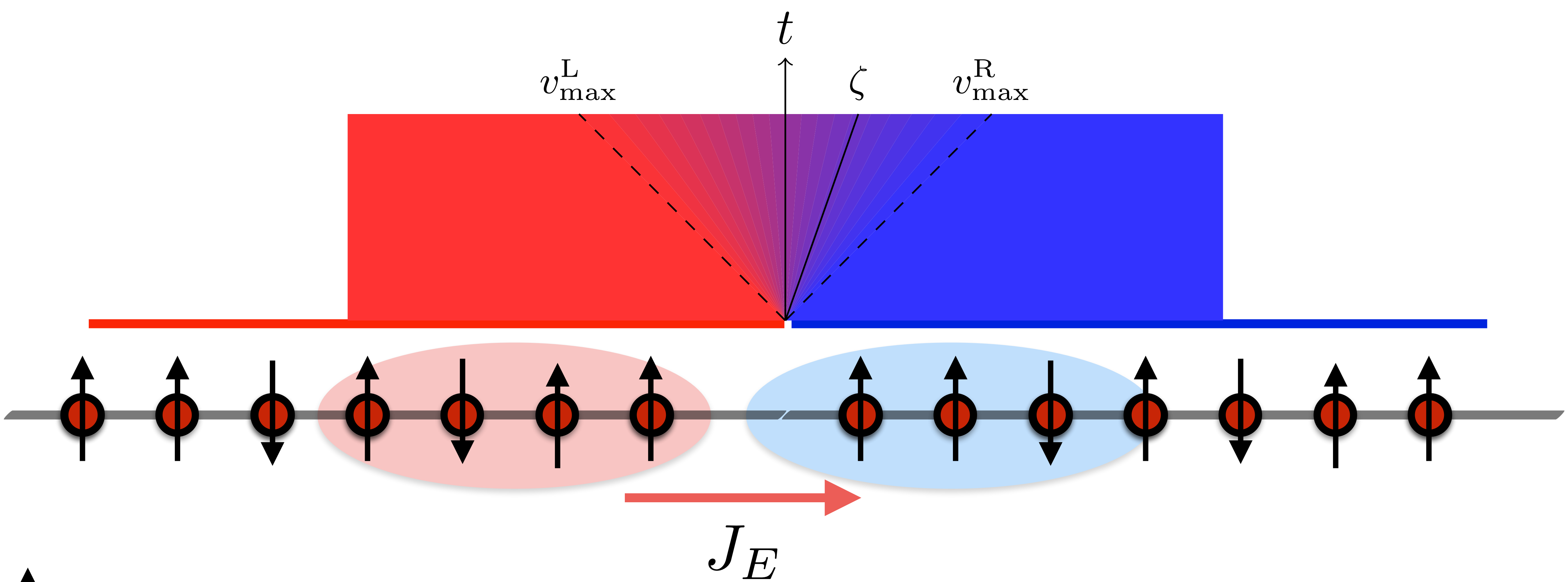


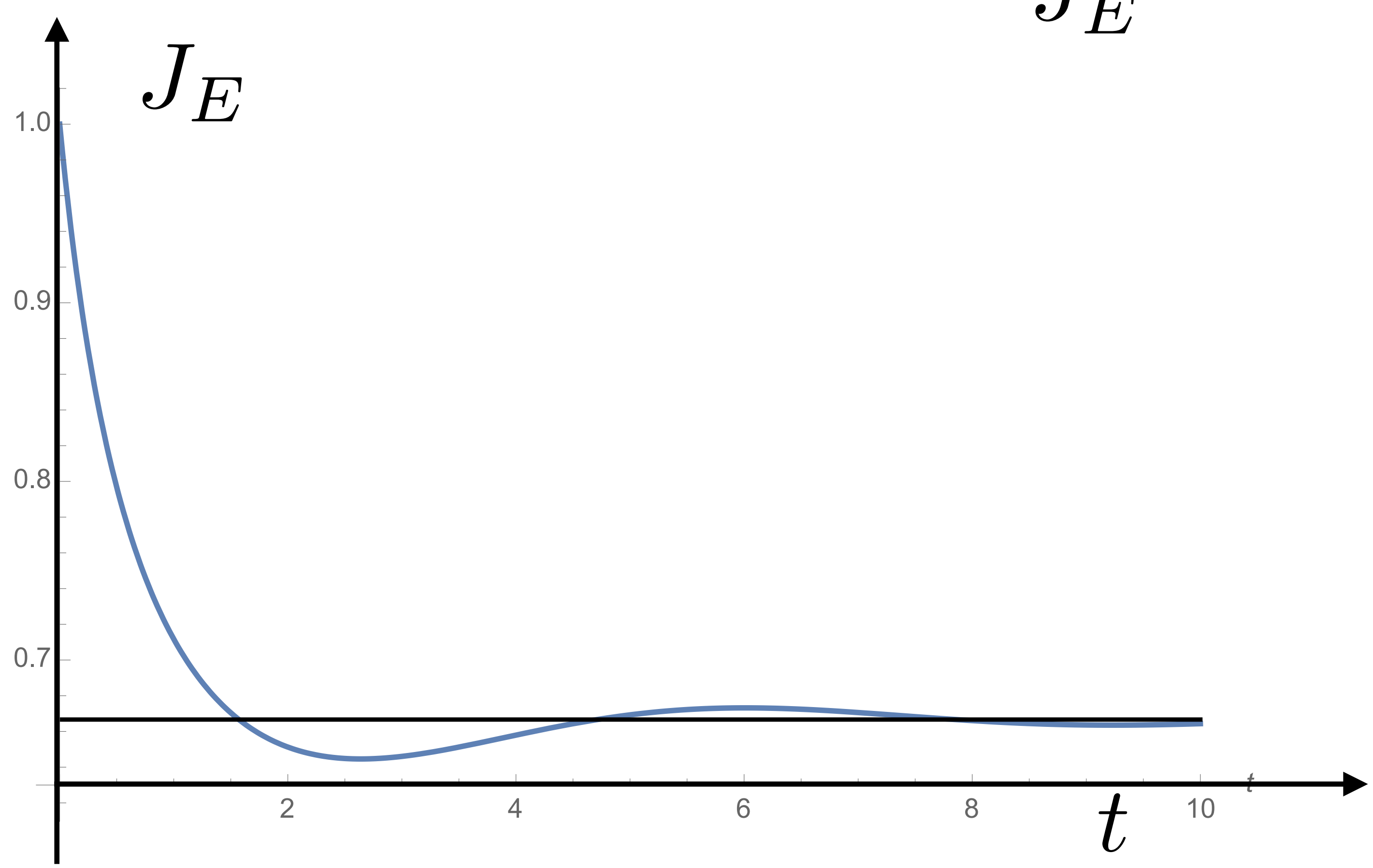
$$\mathrm{Tr}\left(\sigma_1 \sigma_n\, e^{-\beta^{\Psi_0} H - \sum_n \beta_n^{\Psi_0} Q_n}\right)/Z$$

Generalized Gibbs Ensemble (GGE)

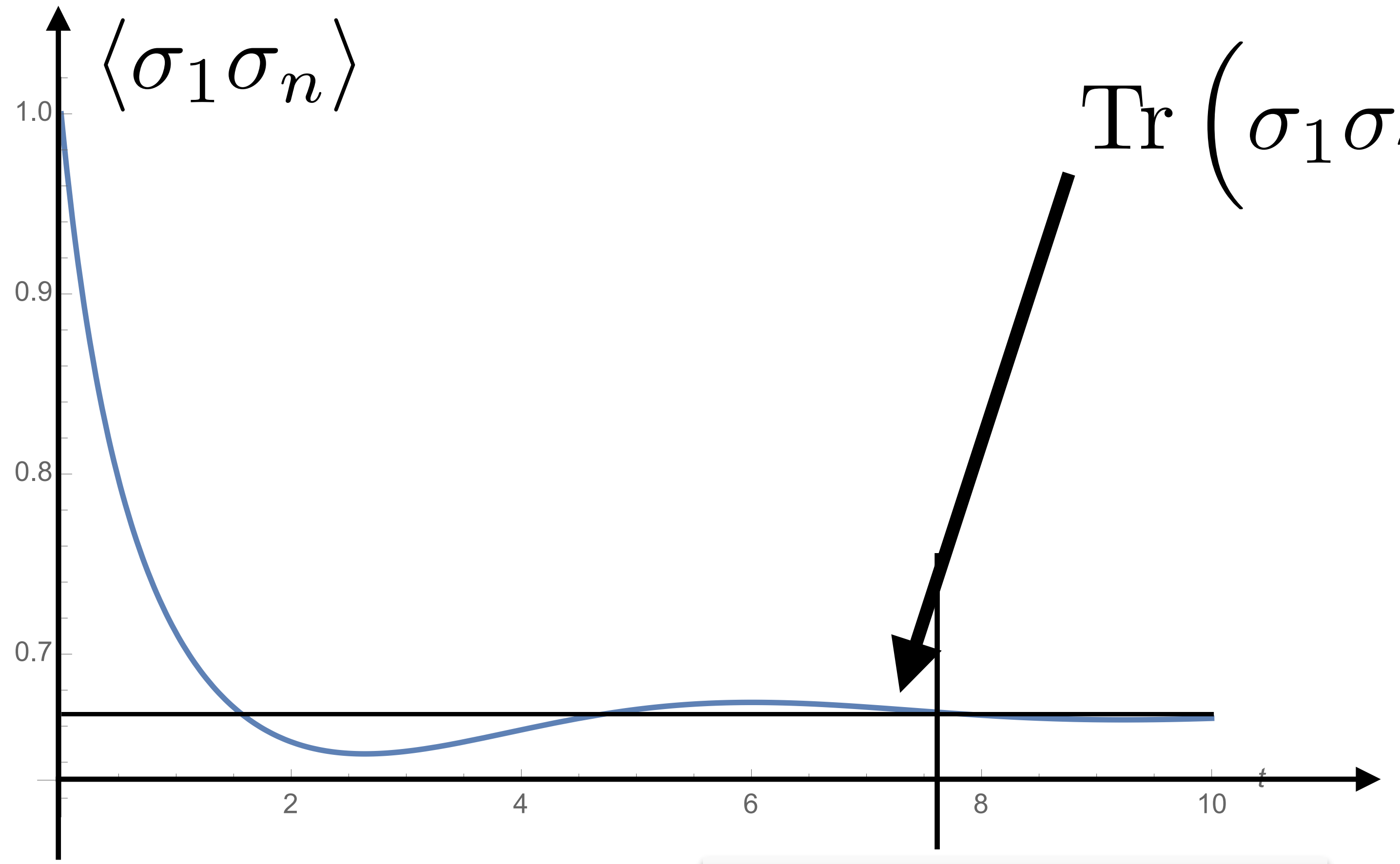
JSTAT Special Issue
ON QUANTUM INTEGRABILITY
IN OUT OF EQUILIBRIUM SYSTEMS

Non-equilibrium dynamics from inhomogeneous states





The quench problem: homogenous state

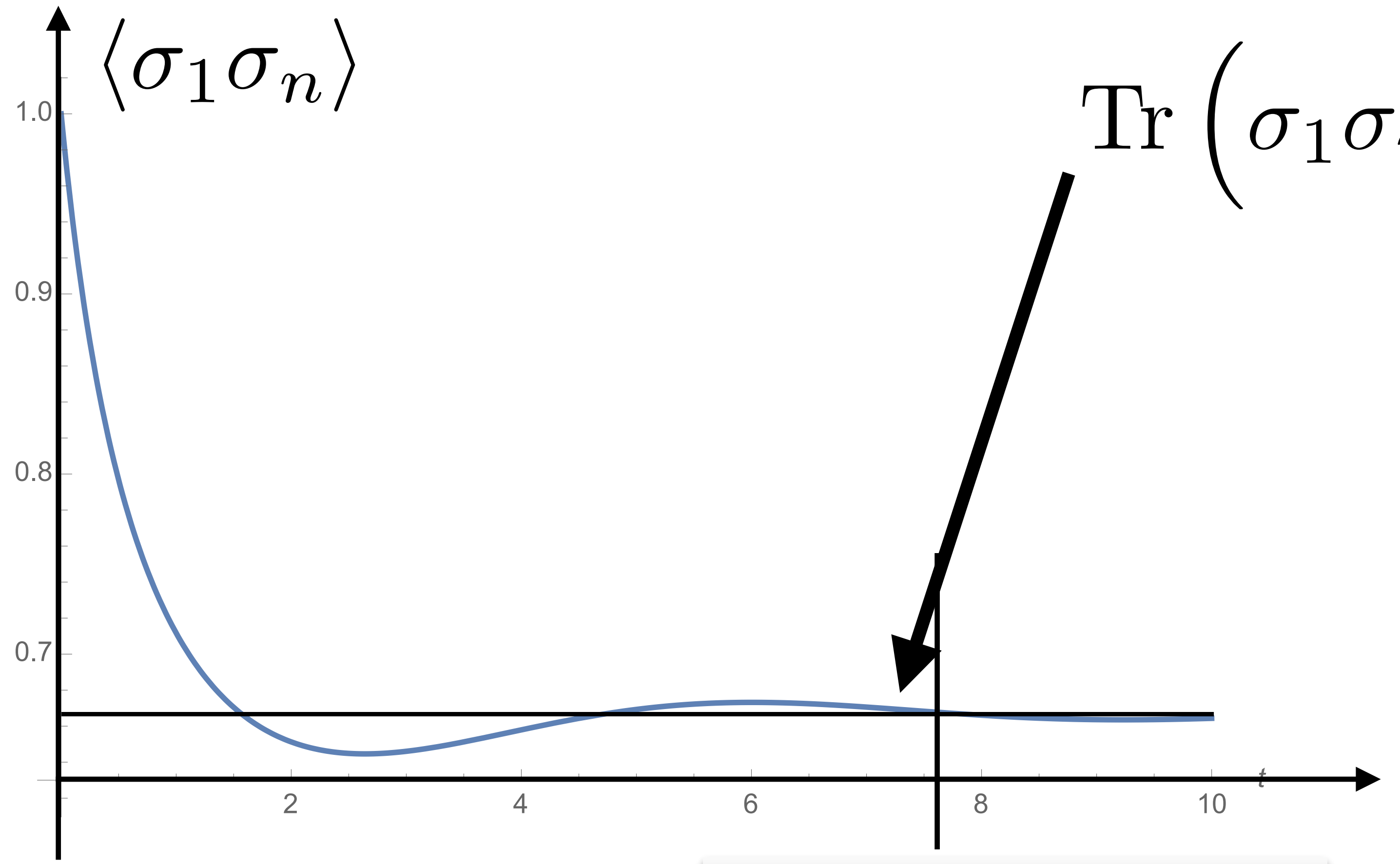


$$\mathrm{Tr}\left(\sigma_1\sigma_n\ e^{-\beta^{\Psi_0}H-\sum_n \beta_n^{\Psi_0}Q_n}\right)/Z$$

GGE TBA macrostate

$$|\Psi_0\rangle \rightarrow \begin{array}{c}\{\rho_n^{\Psi_0}(\lambda)\}_n \\ \{\vartheta_n^{\Psi_0}(\lambda)\}_n\end{array} \rightarrow \langle \vartheta^{\Psi_0}|\hat{O}|\vartheta^{\Psi_0}\rangle$$

The quench problem: homogenous state



$$\mathrm{Tr}\left(\sigma_1\sigma_n\ e^{-\beta^{\Psi_0}H-\sum_n \beta_n^{\Psi_0}Q_n}\right)/Z$$

GGE TBA macrostate

$$|\Psi_0\rangle$$

String-Charge duality

$$\{\rho_n^{\Psi_0}(\lambda)\}_n$$

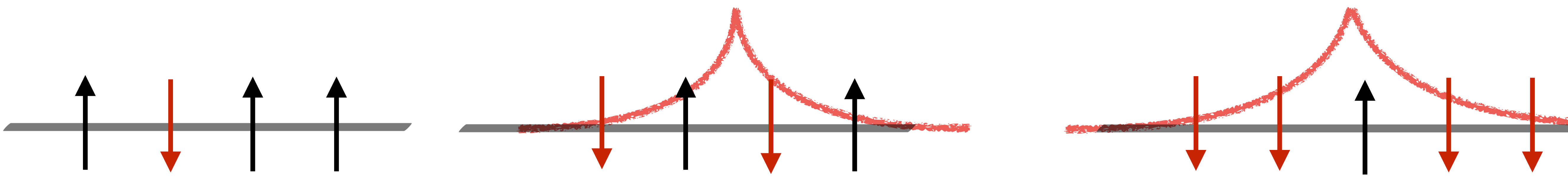
$$\{\vartheta_n^{\Psi_0}(\lambda)\}_n$$

$$\langle\vartheta^{\Psi_0}|\hat{O}|\vartheta^{\Psi_0}\rangle$$

Ilievski, De Nardis et al
PRL (2015)

Ilievski, De Nardis, Brockmann
J Stat. Mech. (2016)

String-charge duality

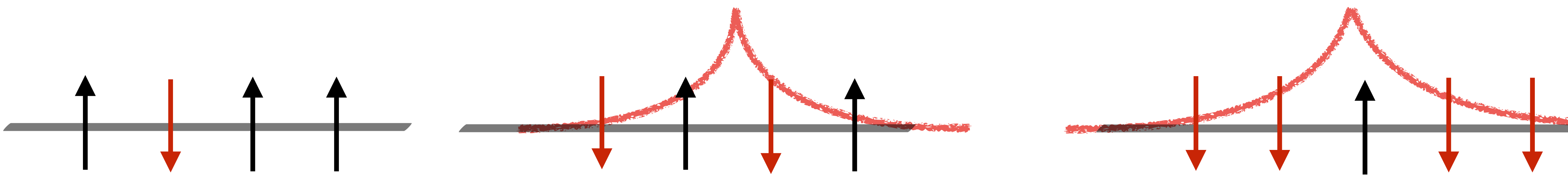


$\rho_1(\lambda)$ $\rho_2(\lambda)$ ………… $\rho_s(\lambda)$

$\Downarrow$ $1:1$

$\langle\Psi_0|\hat{X}_1(\lambda)|\Psi_0\rangle$ $\langle\Psi_0|\hat{X}_2(\lambda)|\Psi_0\rangle$ ………… $\langle\Psi_0|\hat{X}_s(\lambda)|\Psi_0\rangle$

String-charge duality



$\rho_1(\lambda)$ $\rho_2(\lambda)$ .... $\rho_s(\lambda)$

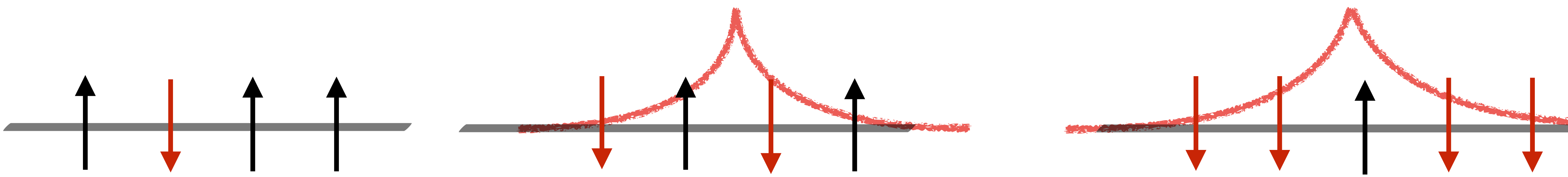
$1:1$

$\langle\Psi_0|\hat{X}_1(\lambda)|\Psi_0\rangle$ $\langle\Psi_0|\hat{X}_2(\lambda)|\Psi_0\rangle$ .... $\langle\Psi_0|\hat{X}_s(\lambda)|\Psi_0\rangle$

$$\hat{X}_s(\lambda) \propto T_s(\lambda - i\eta/2)\partial_\lambda T_s(\lambda + i\eta/2)$$

$s+1$ dimensional representation for the auxiliary spin

String-charge duality



$\rho_1(\lambda)$ $\quad$ $\rho_2(\lambda)$ $\quad \cdots\cdots\cdots$ $\quad$ $\rho_s(\lambda)$

$1:1$

$\langle\Psi_0|\hat{X}_1(\lambda)|\Psi_0\rangle$ $\quad$ $\langle\Psi_0|\hat{X}_2(\lambda)|\Psi_0\rangle$ $\quad \cdots\cdots\cdots$ $\quad$ $\langle\Psi_0|\hat{X}_s(\lambda)|\Psi_0\rangle$

$$\hat{X}_s(\lambda) \propto T_s(\lambda - i\eta/2)\partial_\lambda T_s(\lambda + i\eta/2)$$

$$\hat{X}_s(\lambda) = \sum_{n=0}^{\infty} \frac{\lambda^n}{n!}\hat{Q}_s^{(n)}$$

$s+1$ dimensional representation for the auxiliary spin

Charges and currents

$$\hat{X}_s(\lambda) = \sum_{n=0}^{\infty} \frac{\lambda^n}{n!} \hat{Q}_s^{(n)}$$

$$\partial_t \hat{Q}_s^{(n)} = 0$$

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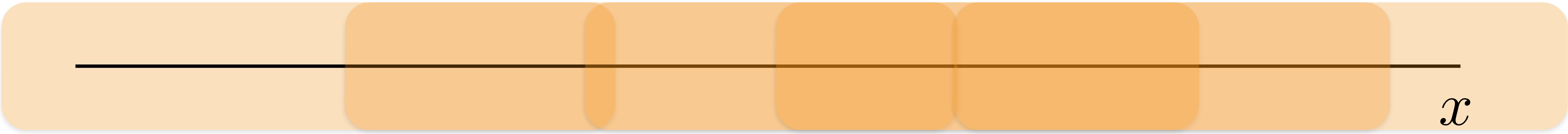

$$\partial_t \hat{Q}_s^{(n)} = -\partial_x \hat{J}_s^{(n)}$$

Charges and currents

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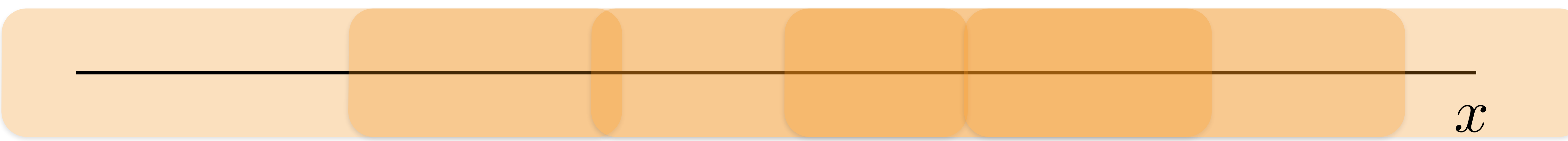
$$\langle \vartheta | \hat{Q}_s^{(n)} | \vartheta \rangle = \sum_a \int d\lambda \; \rho_a(\lambda) q_{a,s}^{(n)}(\lambda)$$



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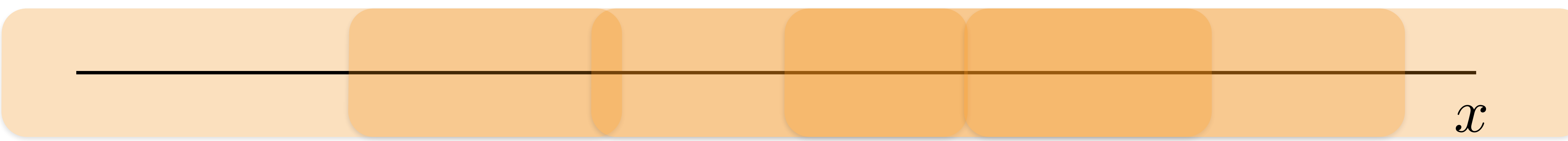


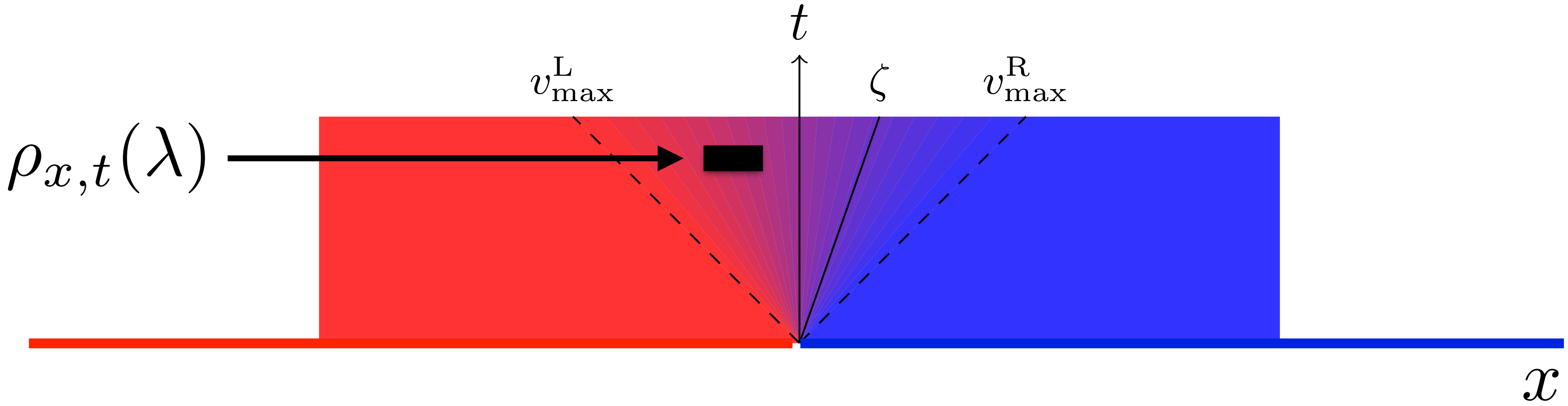
$$\partial_t \hat{Q}_s^{(n)} = -\partial_x \hat{J}_s^{(n)}$$

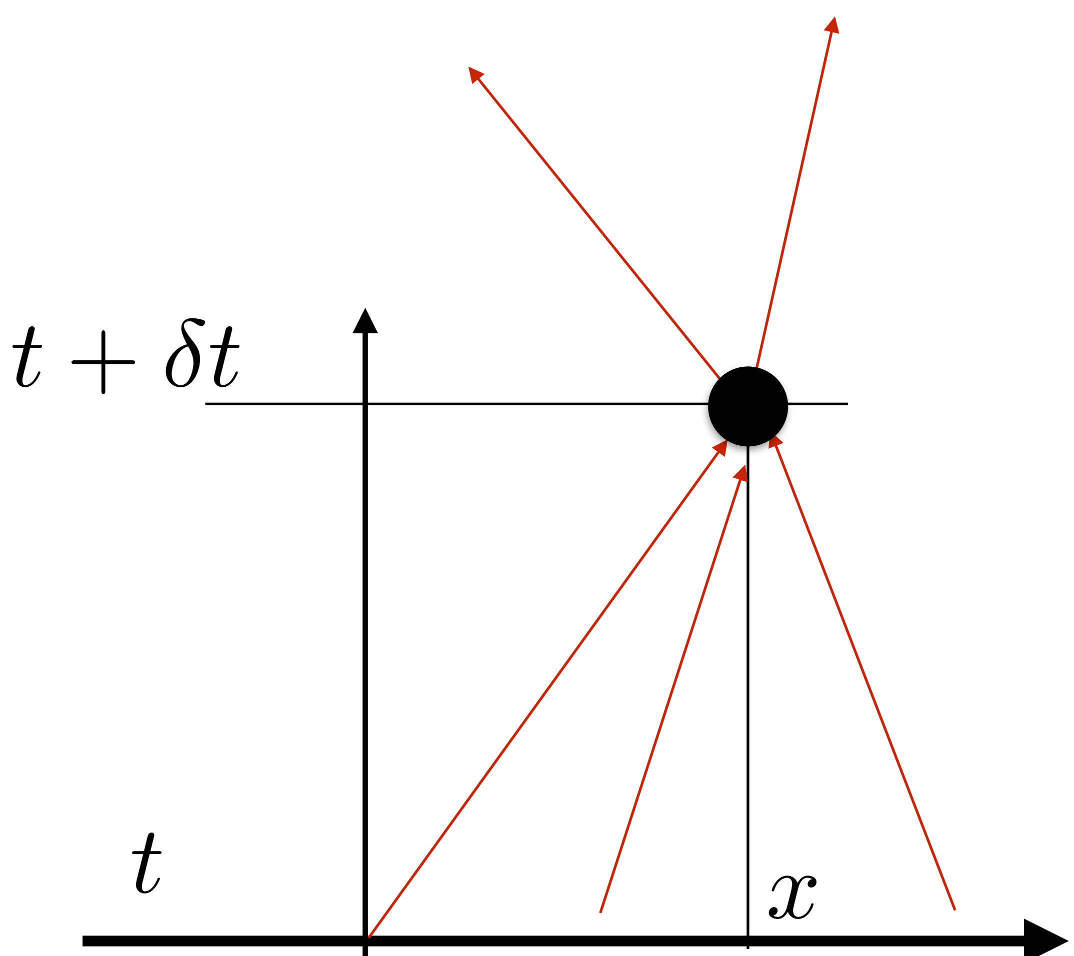
Castro-Alvaredo, Yoshimura, Doyon, PRX (2016)

Bertini, Collura, De Nardis, Fagotti, PRL (2016)

$$\langle \vartheta | \hat{J}_s^{(n)} | \vartheta \rangle = \sum_a \int d\lambda \; \Big[\rho_a(\lambda) v_a(\lambda) \Big] q_{a,s}^{(n)}(\lambda)$$

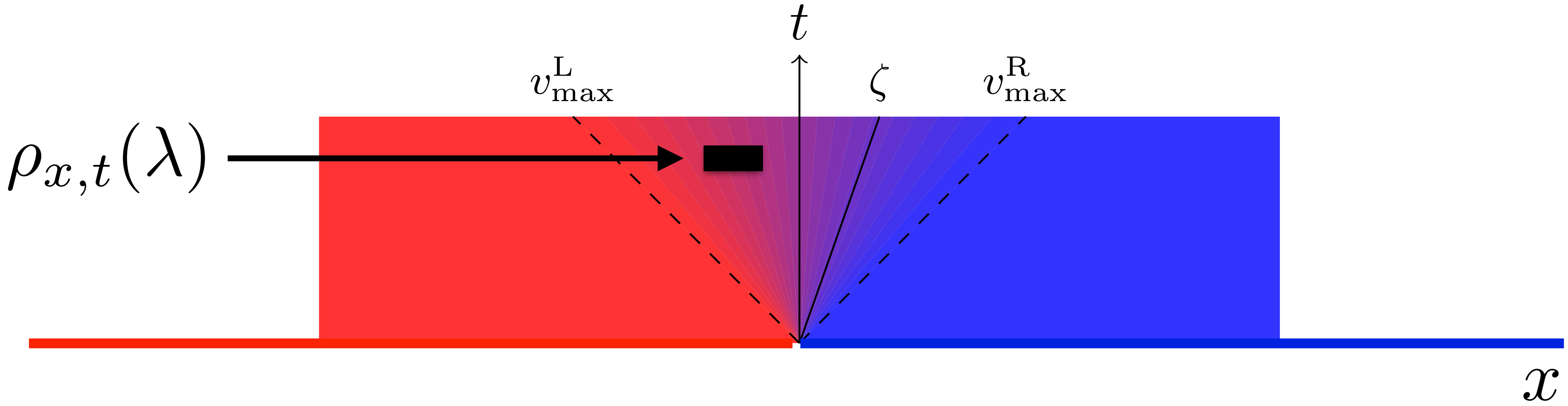
Hydrodynamics: continuity equation



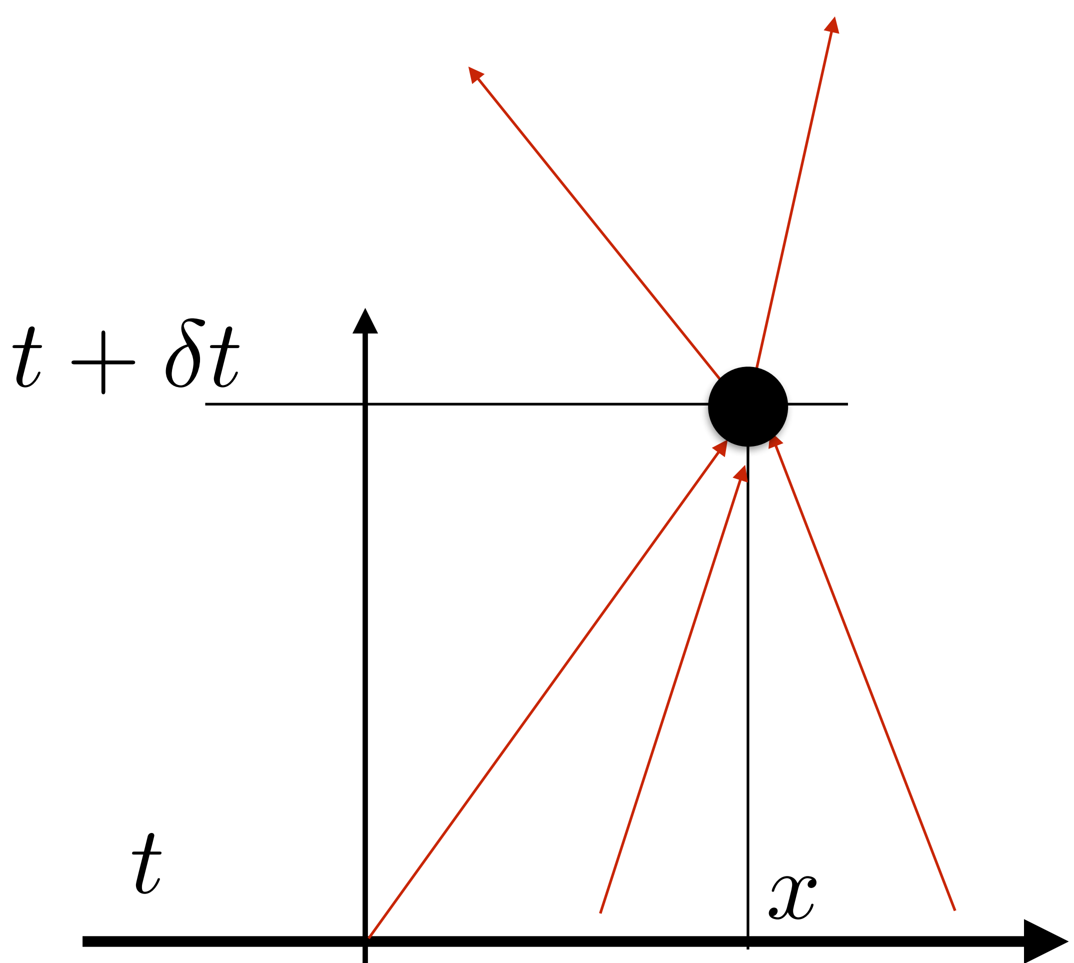


$$v_{x,t}(\lambda) = \frac{\partial \varepsilon_{x,t}(\lambda)}{\partial k_{x,t}(\lambda)}$$

Hydrodynamics: continuity equation



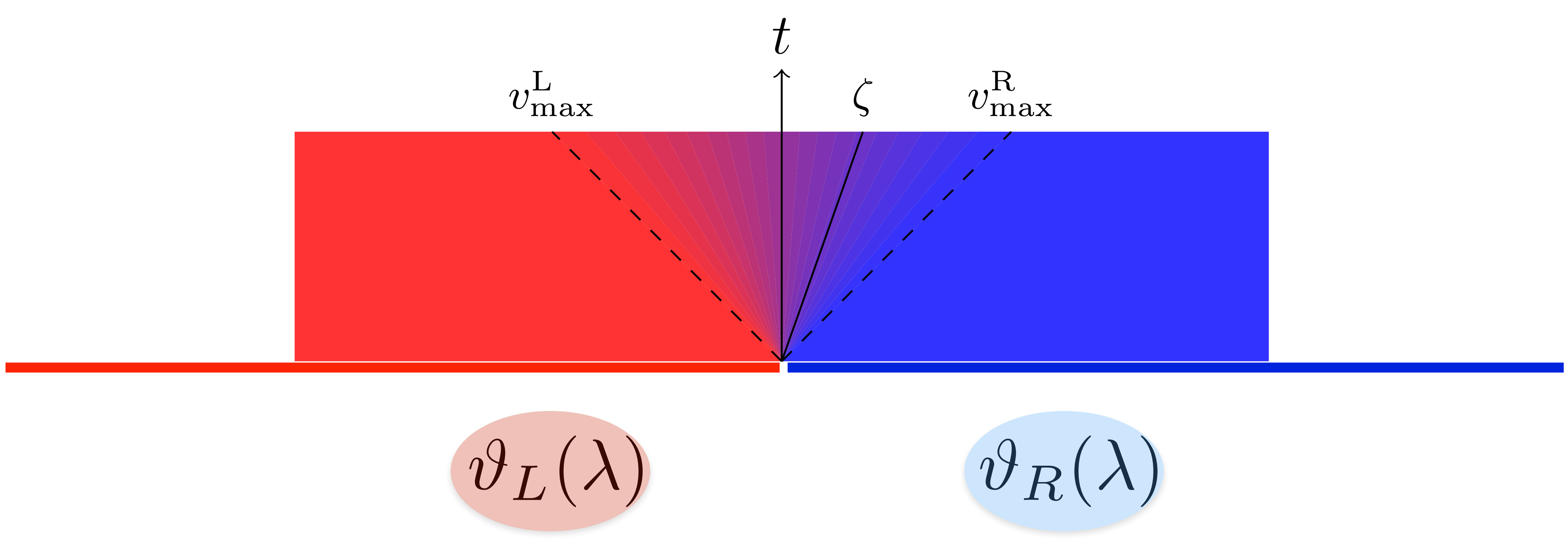
$$\partial_t \rho_{x,t} = -\partial_x \left(v_{x,t} \rho_{x,t} \right)$$



$$v_{x,t}(\lambda) = \frac{\partial \varepsilon_{x,t}(\lambda)}{\partial k_{x,t}(\lambda)}$$

Hydrodynamics: solution of the bi-partite problem

$$\partial_t \rho_{x,t} = -\partial_x \left(v_{x,t} \rho_{x,t} \right)$$

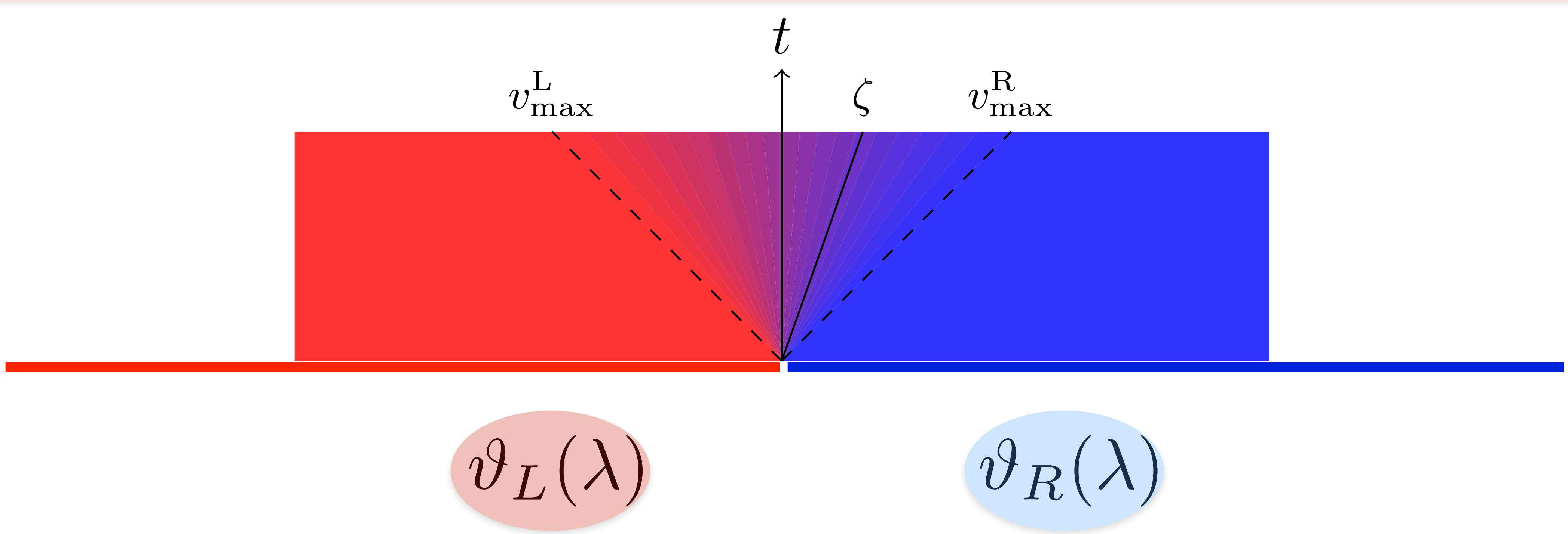


Hydrodynamics: solution of the bi-partite problem

$$\partial_t \rho_{x,t} = -\partial_x \left(v_{x,t} \rho_{x,t} \right)$$

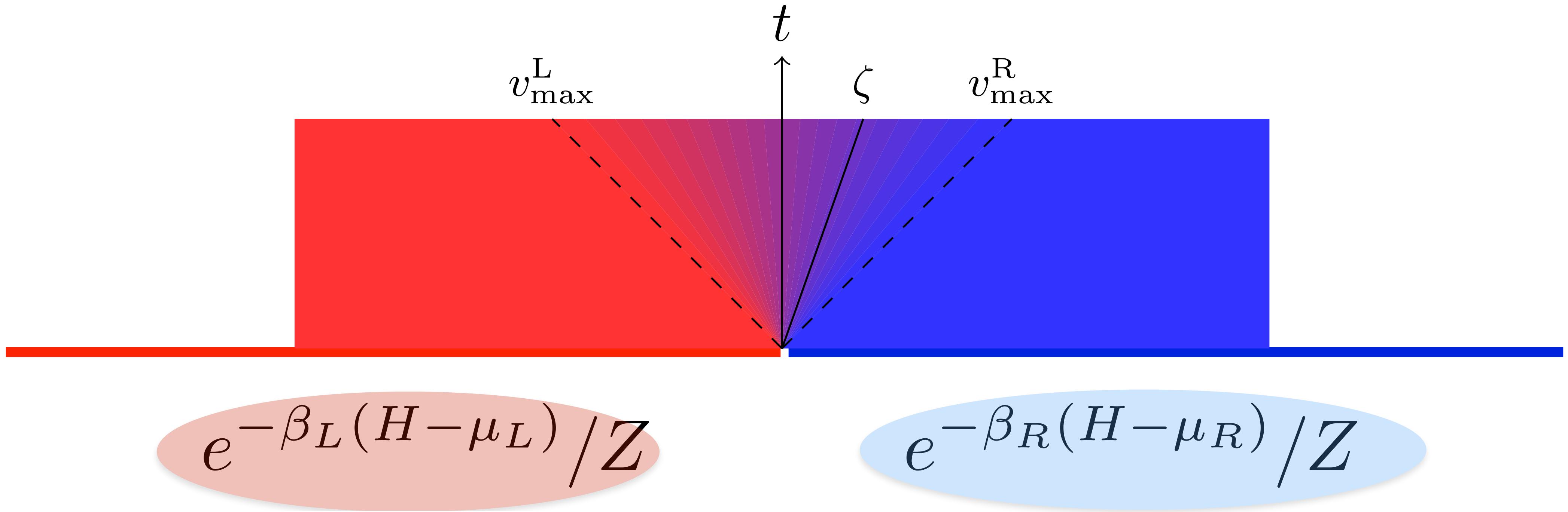
$t \to \infty$ $\quad \rho_{x,t} \to \rho_\zeta$ $\quad \vartheta_{x,t} \to \vartheta_\zeta$ $\quad \zeta = x/t$

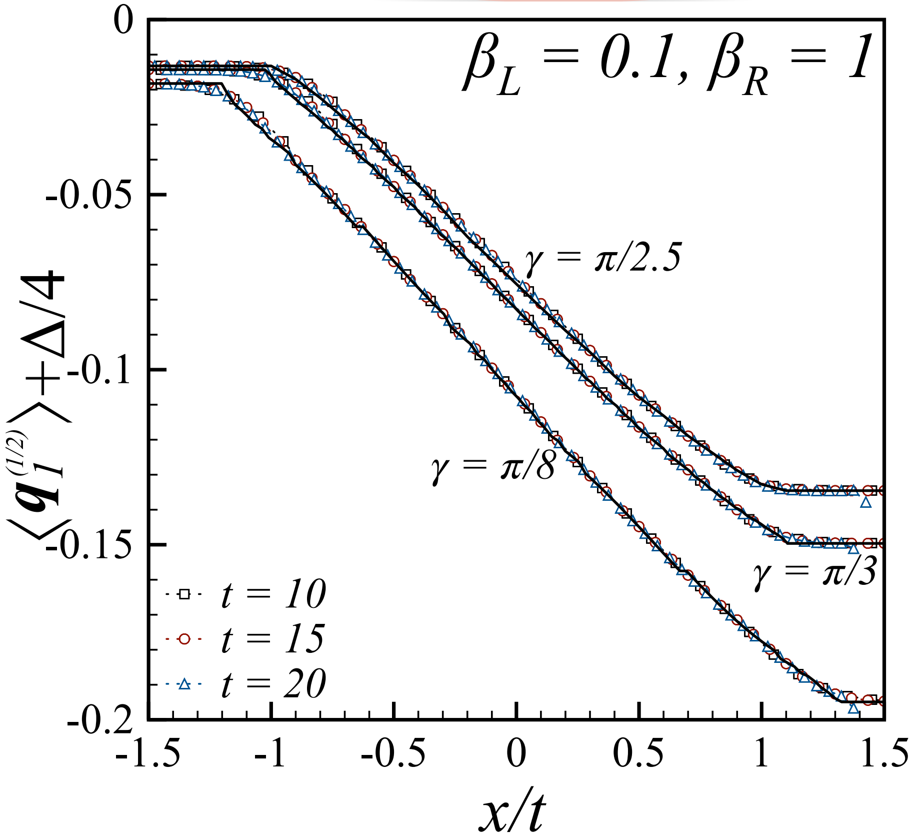
$$\vartheta_\zeta(\lambda) = \begin{cases} \vartheta_L(\lambda) \text{ if } v_\zeta(\lambda) \geq \zeta \\ \vartheta_R(\lambda) \text{ if } v_\zeta(\lambda) < \zeta \end{cases}$$

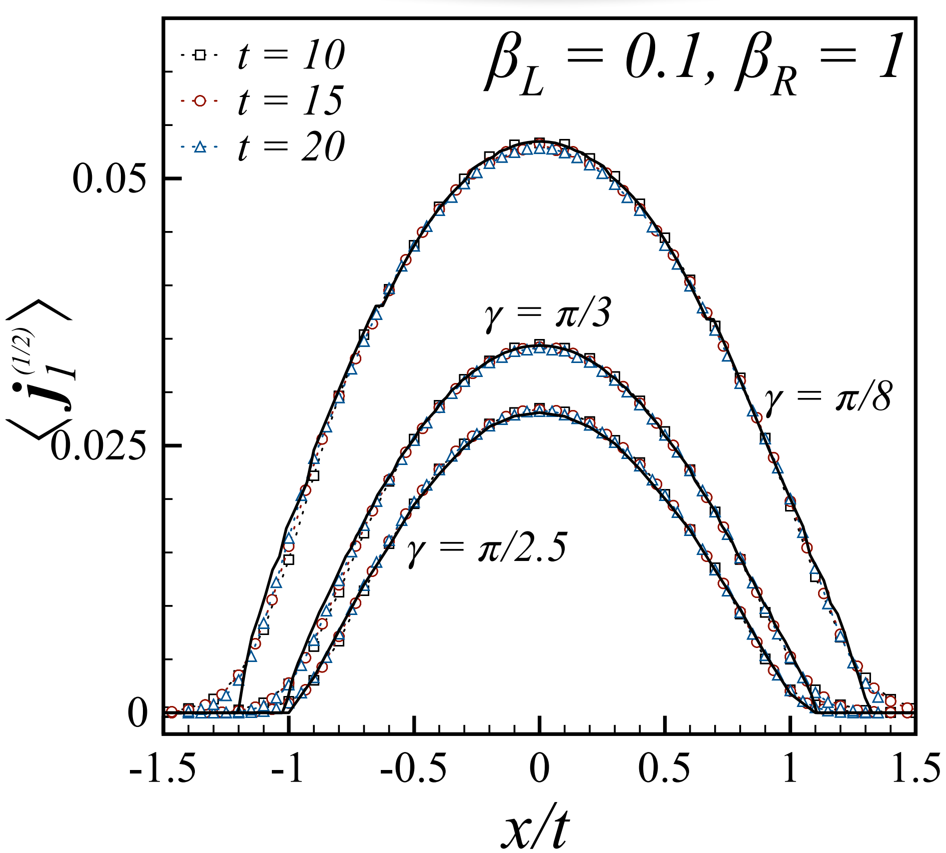


Hydrodynamics

$$H = \frac{J}{4}\sum_{j=1}^{N}\left(\sigma_j^x\sigma_{j+1}^x + \sigma_j^y\sigma_{j+1}^y + \Delta(\sigma_j^z\sigma_{j+1}^z - 1)\right)$$



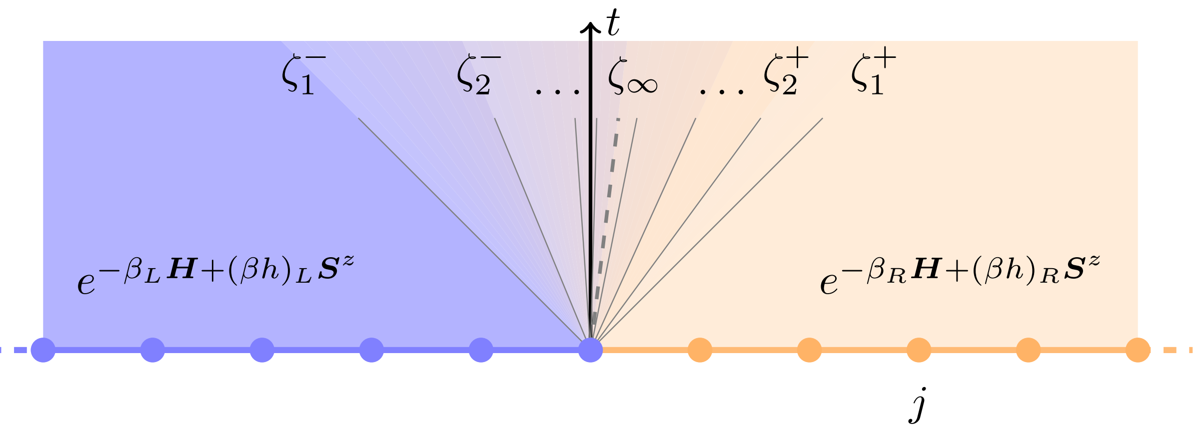




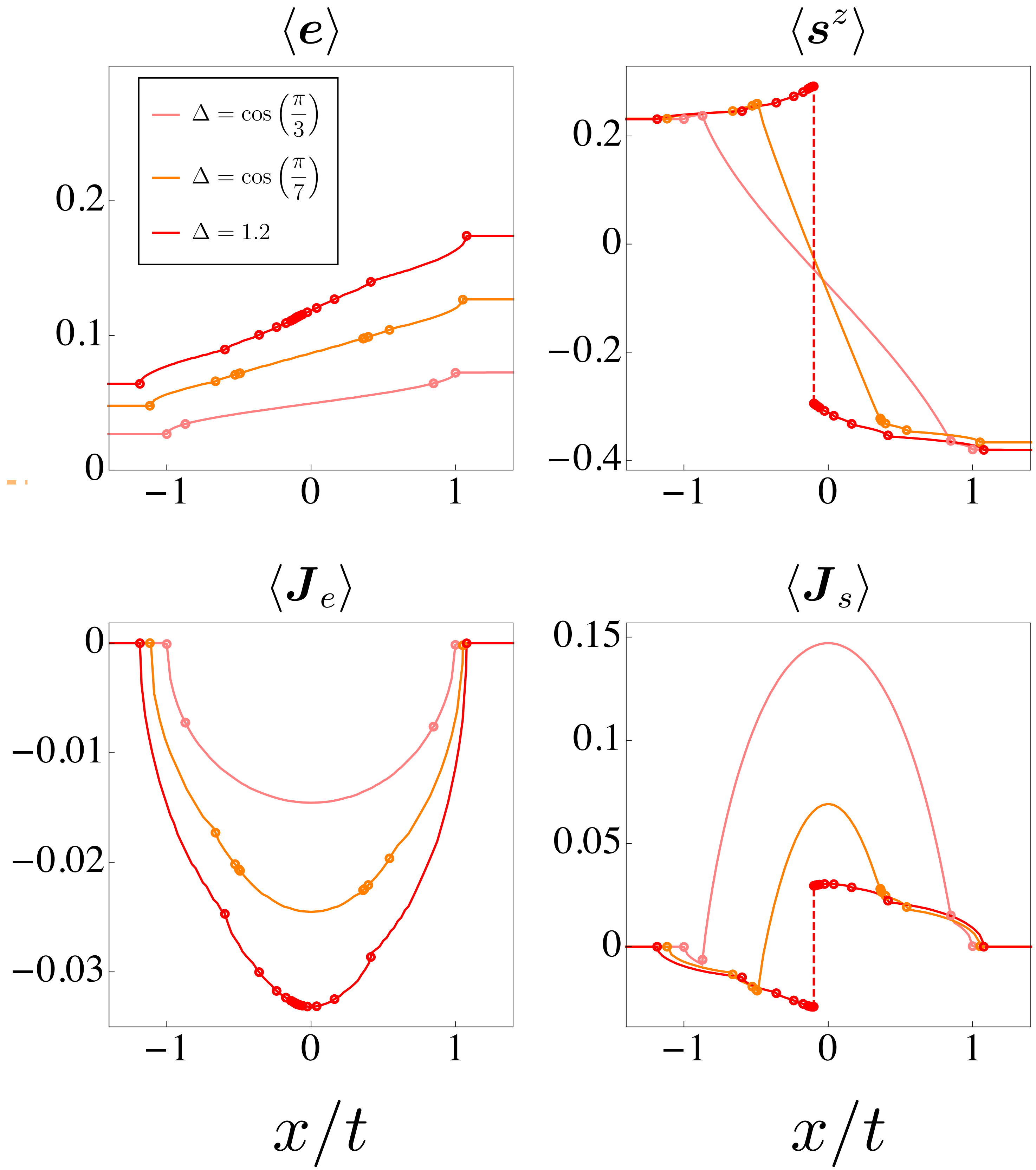
Hydrodynamics: light-cones and magnetization jumps

Piroli, De Nardis et al
PRB (2017)

De Luca, Collura, De Nardis
PRB (2017)



$\beta_L = 0, (\beta h)_L = 1 \quad \beta_R = 0, (\beta h)_R = -2$



Hydrodynamics and correlation functions

GGE state

$$\{\rho_n(\lambda)\}_n$$

$$\{\vartheta_n(\lambda) = \frac{\rho_n(\lambda)}{2\pi k'_n(\lambda)}\}_n$$

$$\Rightarrow \quad \langle\vartheta|\sigma_1(0)\sigma_n(t)|\vartheta\rangle$$

What can hydrodynamics teach us about the dynamical correlations of a generic eigenstate?

Hydrodynamic regime: matrix elements

$$\langle \hat{q}(x,t)\hat{q}(0,0)\rangle = \int dx dt \; e^{ikx-i\omega t} S_q(k,\omega)$$

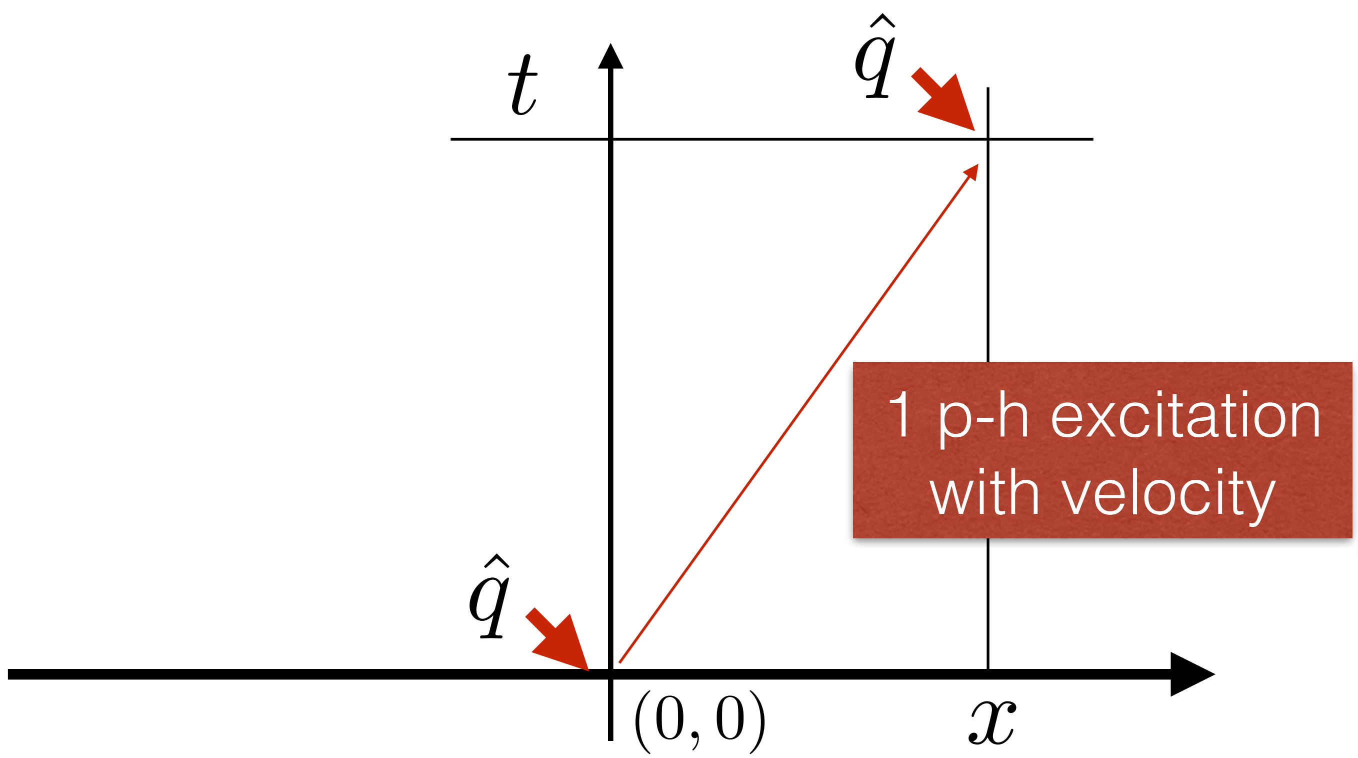
$$(x,t) \gg 1 \quad \rightarrow \int dh \; \delta(x - v(h)t) P_{h\rightarrow h}$$

$$\hat{Q} = \int dx \hat{q}(x)$$

$$[\hat{Q}, H] = 0$$

$$\hat{q}(x) = \hat{\rho}(x)$$

$$\hat{q}(x) = \sigma_j^z$$



$$v(h) = \frac{\omega}{k}$$

Doyon and Spohn arXiv (2017)

Ilievski and De Nardis PRB (2017)

De Nardis and Panfil SciPost (2016)

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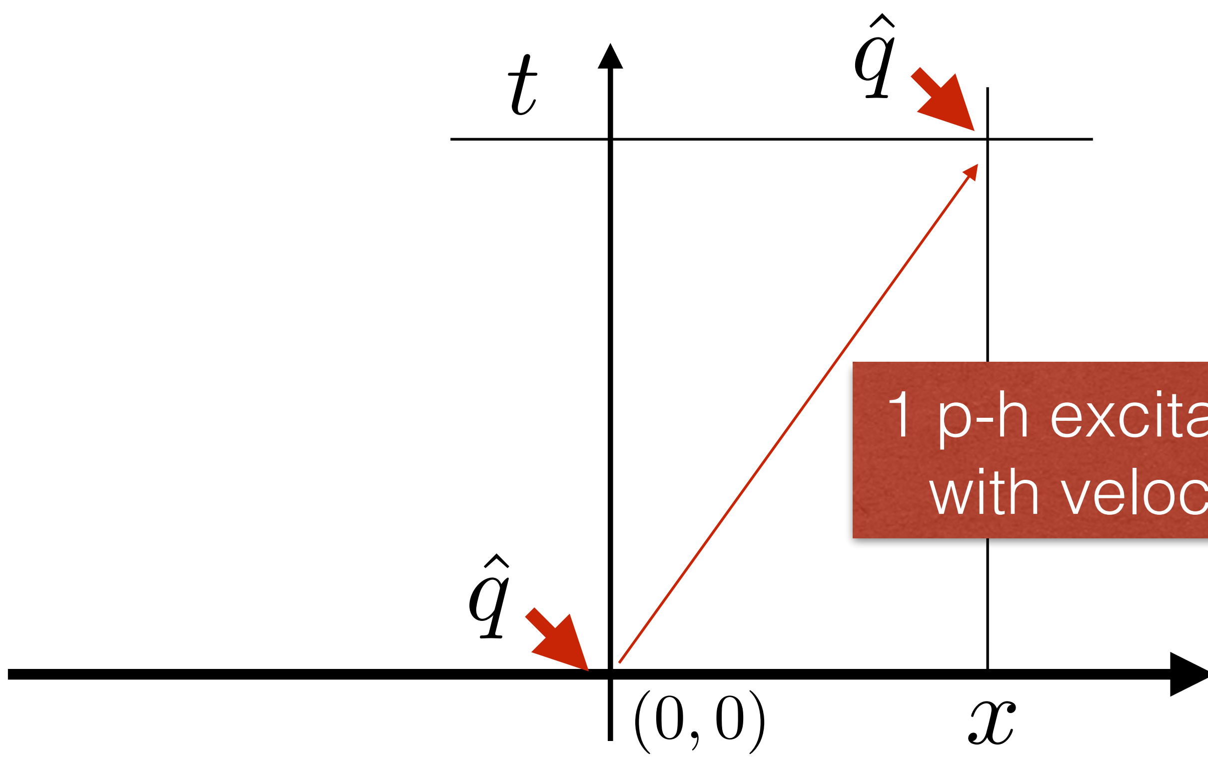
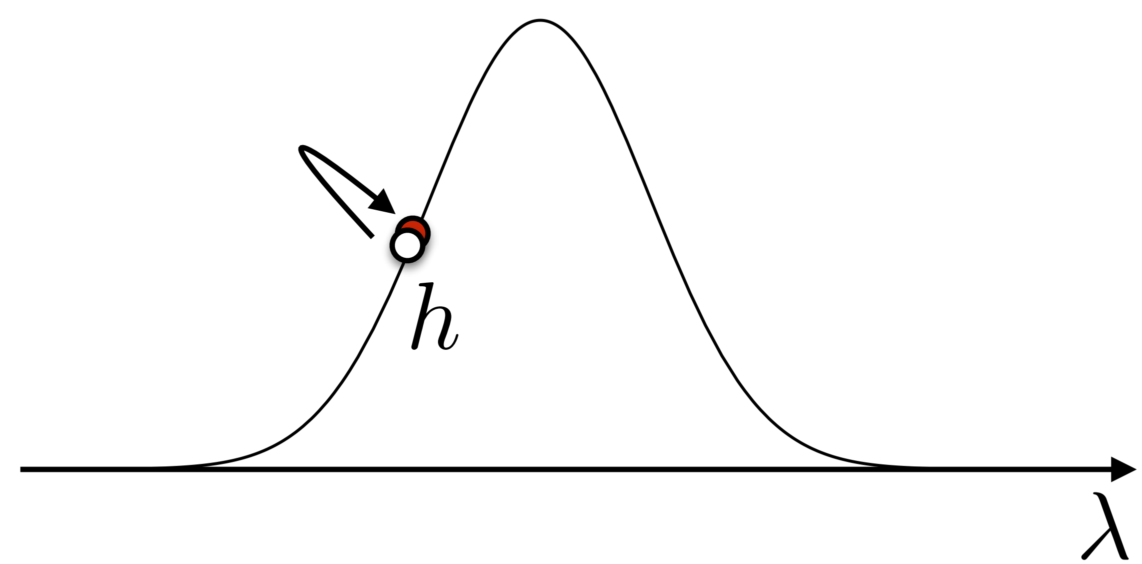


1 p-h excitation with velocity

$$v(h) = \frac{\omega}{k}$$



Doyon and Spohn arXiv (2017)

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De Nardis and Panfil SciPost (2016)

Hydrodynamic regime: matrix elements

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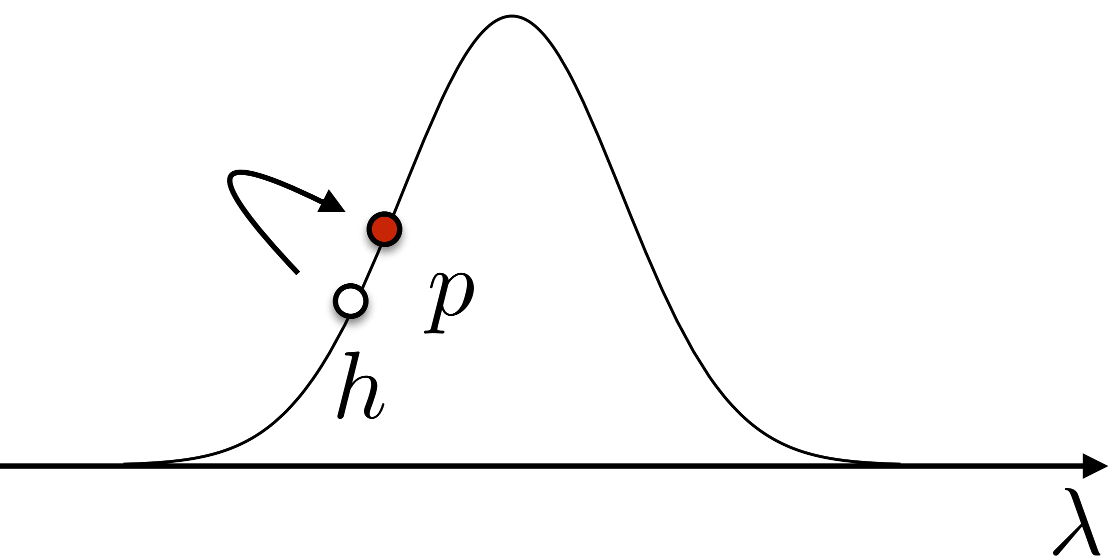
$$(x,t) \gg 1 \quad \to \int dh\ \delta(x - v(h)t) P_{h\to h}$$

$$P_{h\to q} = \rho(h)(1-\vartheta(p))|\langle\vartheta|\hat{q}|\vartheta, h\to p\rangle|^2$$



Dynamical correlations at low momentum are

Given by 1 particle-hole excitations

Matrix elements given by

$$\lim_{h\to p} |\langle\vartheta|\hat{q}|\vartheta, h\to p\rangle| = q^{dr}(h) + O(p-h)$$

$$q^{dr}(h) = \left(1 + \frac{K}{2\pi}\vartheta\right)^{-1}_{h,u} q(u)$$

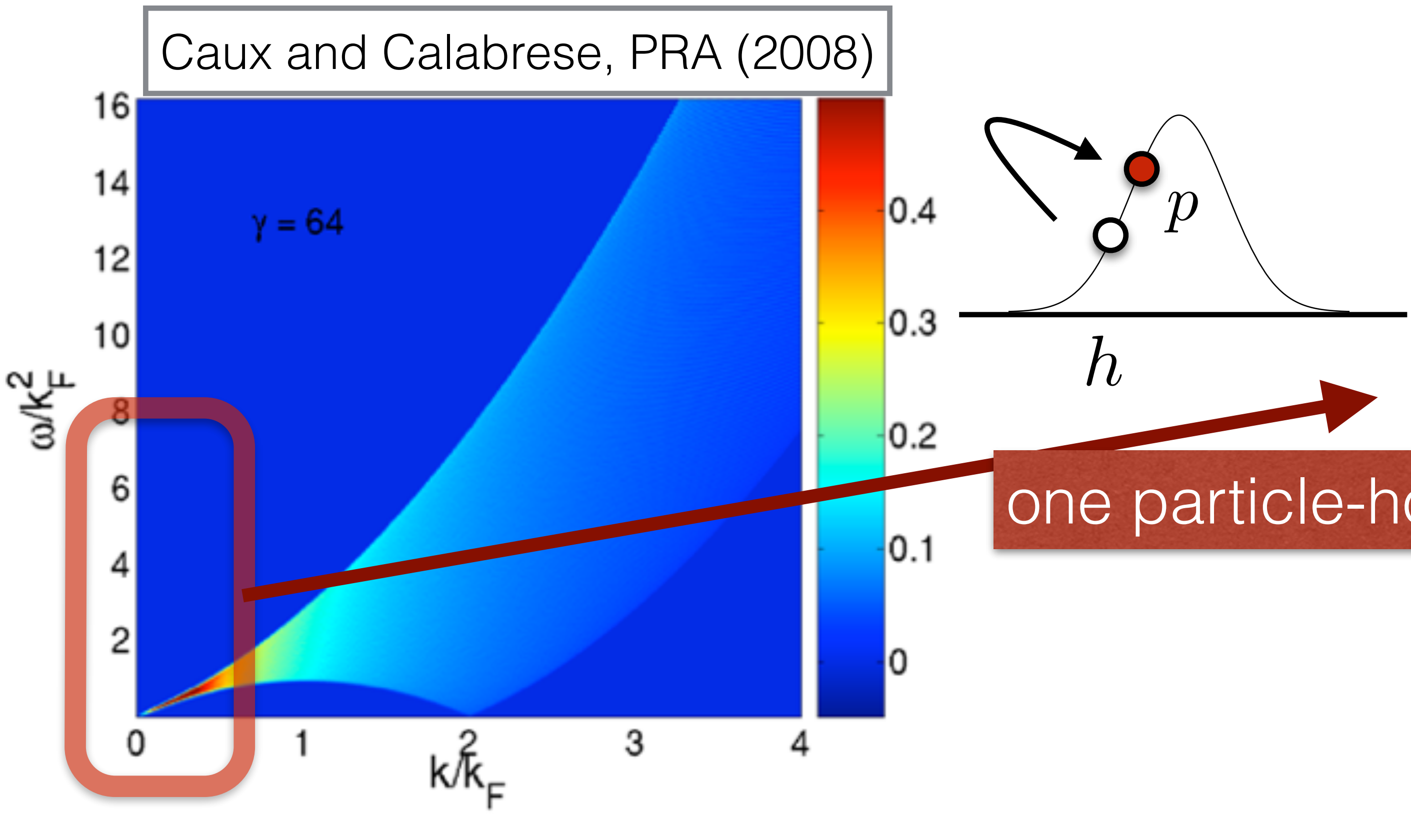
Doyon and Spohn arXiv (2017)

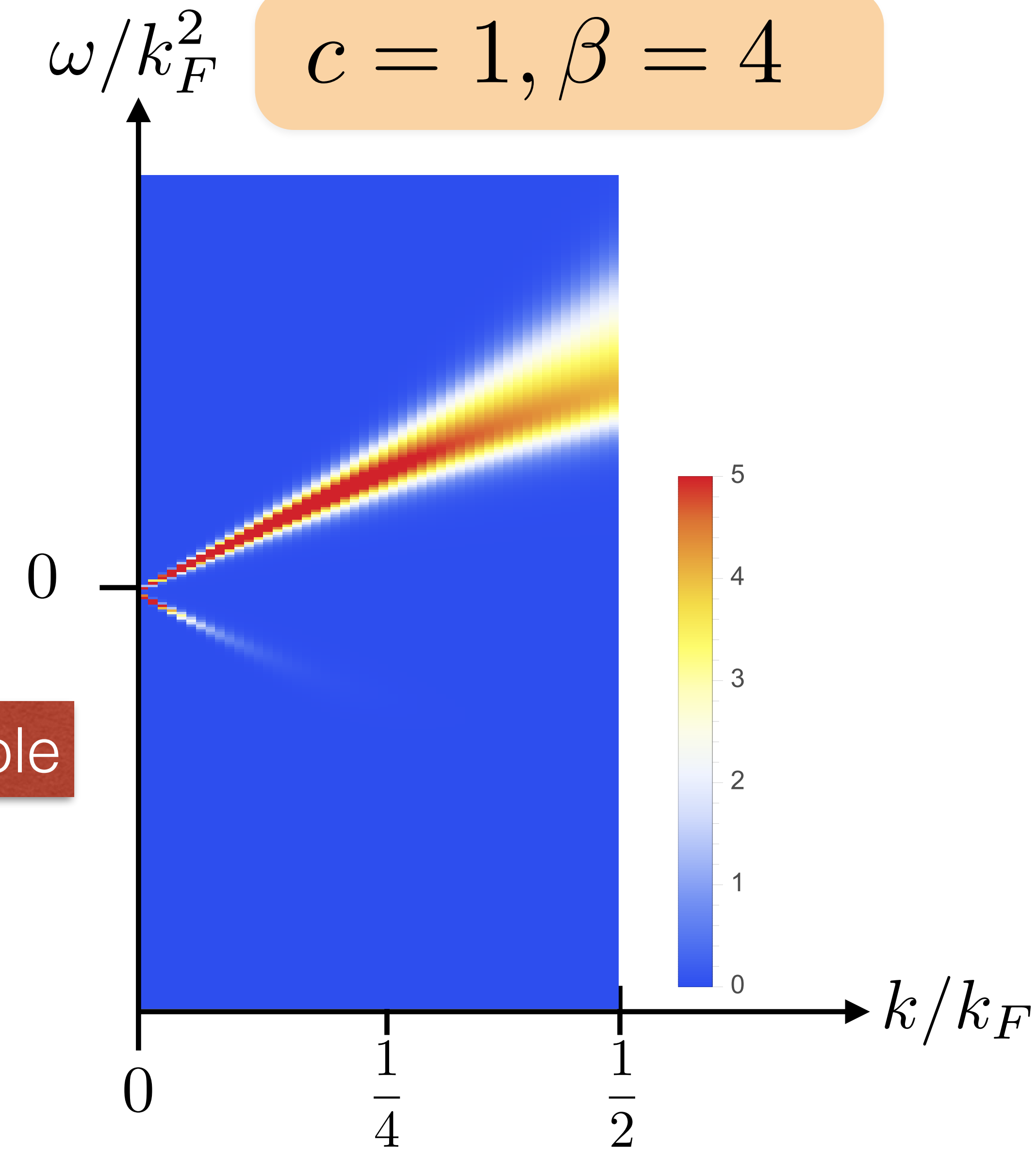
Ilievski and De Nardis PRB (2017)

De Nardis and Panfil SciPost (2016)

Hydrodynamic regime: DSF of the Lieb-Liniger gas

$$\langle \hat{\rho}(x,t)\hat{\rho}(0,0)\rangle = \int dxdt\; e^{ikx-i\omega t} S(k,\omega)$$





$$S_{1ph}(k,\omega) = \rho(h)\rho_h(h+k/k'(h)) \frac{|\langle\vartheta|\hat{\rho}|\vartheta, h\to h+k/k'(h)\rangle|^2}{k'(h)v'(h)k}\Big|_{kv(h+k/(2k'(h)))=\omega}$$

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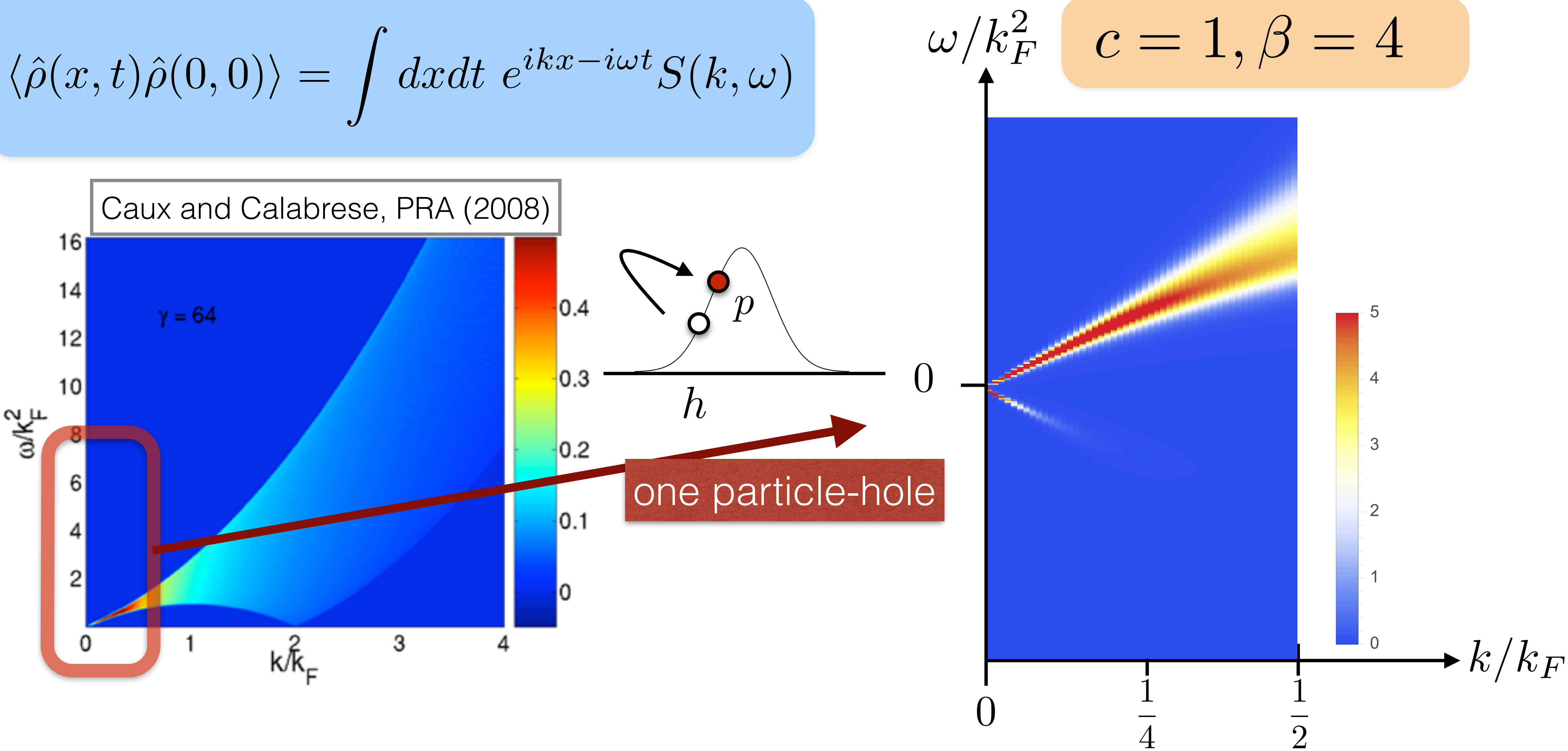

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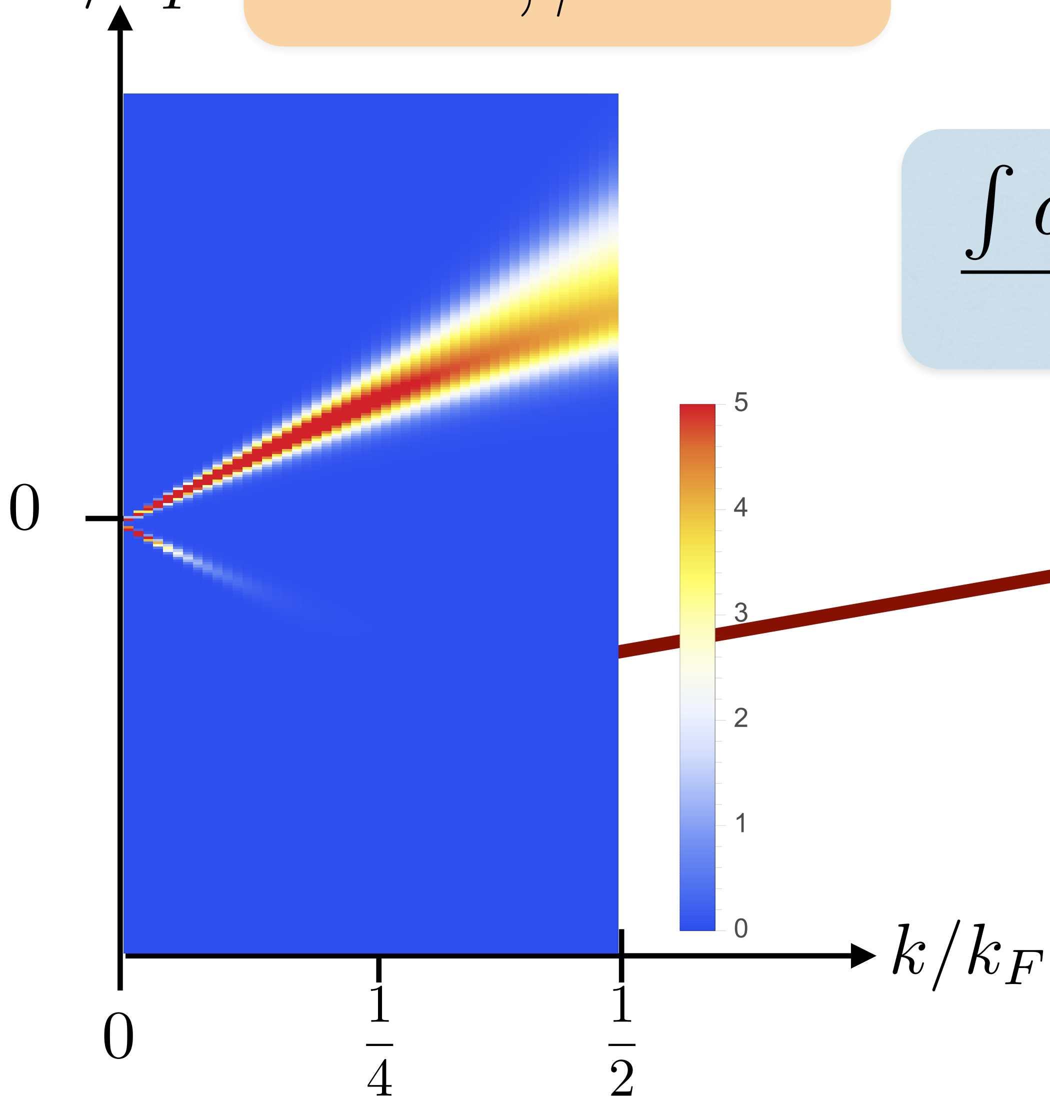
$$\langle\vartheta|\hat{\rho}|\vartheta, h \to p\rangle = \lim_{\text{th}} L \frac{\langle\{\lambda_j\}^\vartheta|\hat{\rho}|\{\mu_j\}^\vartheta\rangle}{||\{\lambda_j\}^\vartheta||\ ||\{\mu_j\}^\vartheta||} \times e^{S_\mu - S_\lambda}$$

Hydrodynamic regime: DSF of the Lieb-Liniger gas

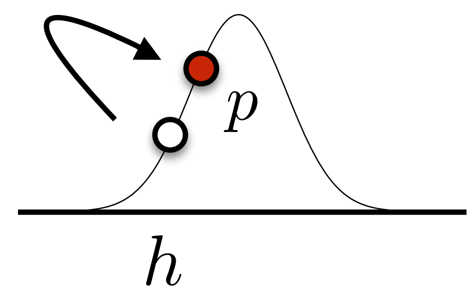
De Nardis and Panfil SciPost (2016)

$c = 1, \beta = 4$

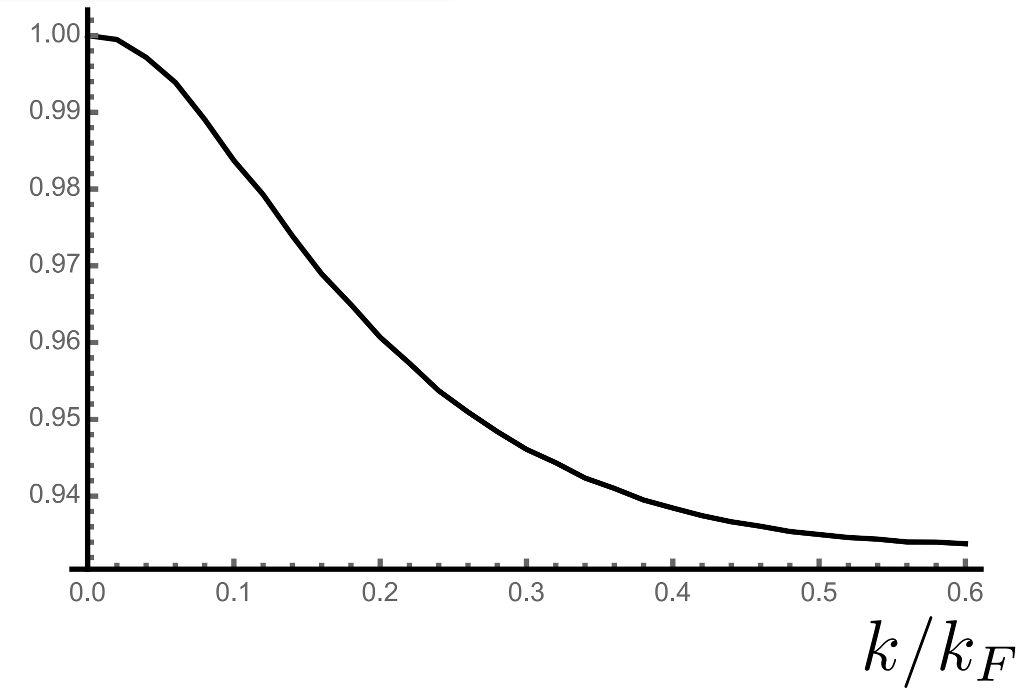


f-sum rule saturated

$$\frac{\int d\omega \omega S_{1ph}(k,\omega)}{2\pi k^2} = 1 - O(k^2)$$



one particle-hole



$$S_{1ph}(k,\omega) = \rho(h)\rho_h(h+k/k'(h)) \frac{|\langle\vartheta|\hat{\rho}|\vartheta, h\to h+k/k'(h)\rangle|^2}{k'(h)v'(h)k}\Big|_{kv(h+k/(2k'(h)))=\omega}$$

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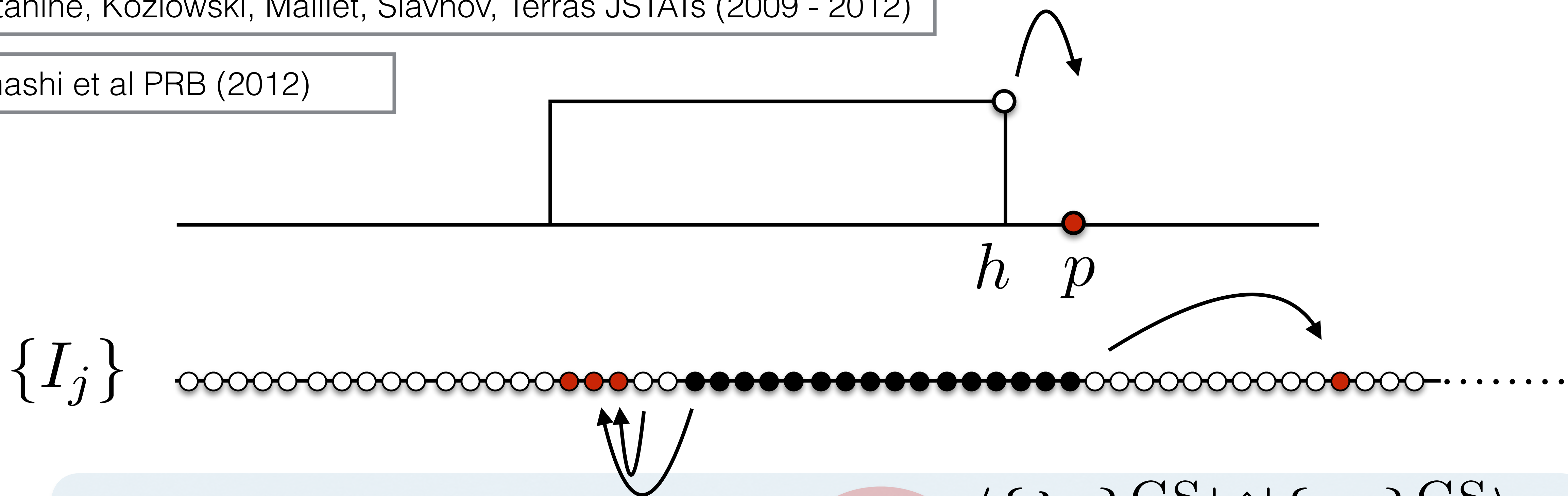
Comparison with results on the ground state

Kitanine, Kozlowski, Maillet, Slavnov, Terras JSTATs (2009 - 2012)

Shashi et al PRB (2012)

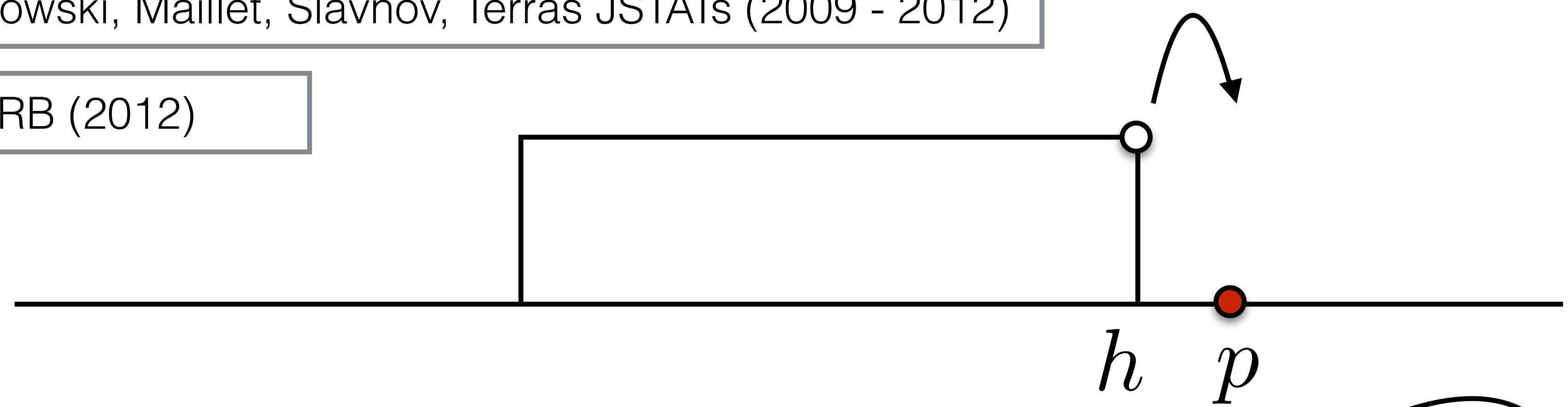


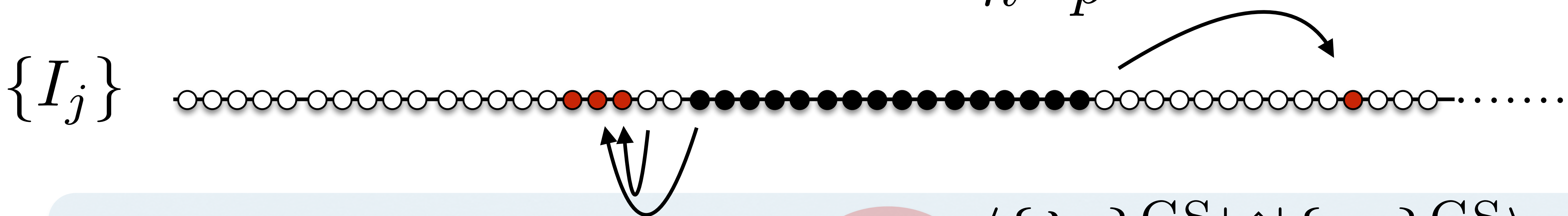
$$\langle \mathrm{GS}|\hat{\rho}|\mathrm{GS}, h \to p\rangle = \lim_{\mathrm{th}} L^{1+\alpha} \frac{\langle \{\lambda_j\}^{\mathrm{GS}}|\hat{\rho}|\{\mu_j\}^{\mathrm{GS}}\rangle}{||\{\lambda_j\}^{\mathrm{GS}}||\ ||\{\mu_j\}^{\mathrm{GS}}||}$$

Comparison with results on the ground state

Kitanine, Kozlowski, Maillet, Slavnov, Terras JSTATs (2009 - 2012)

Shashi et al PRB (2012)





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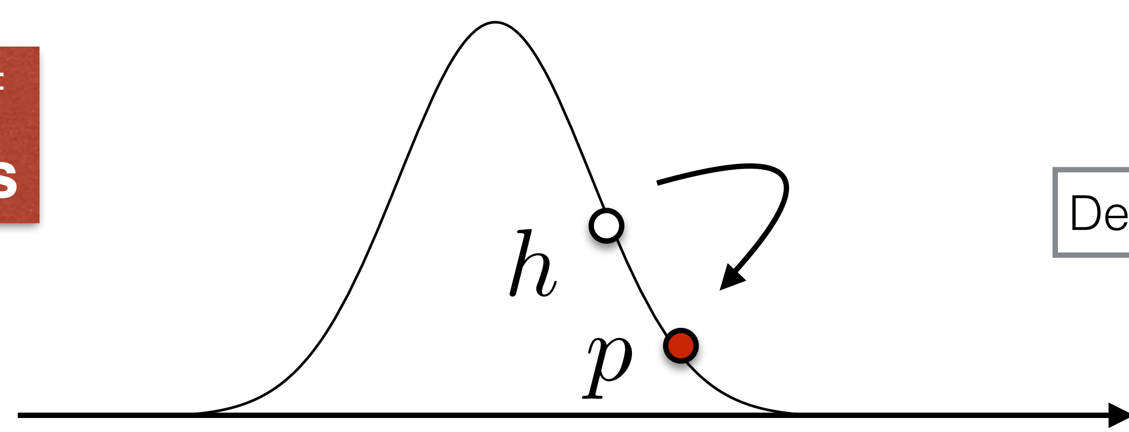
Excitations on top of **finite entropy states**



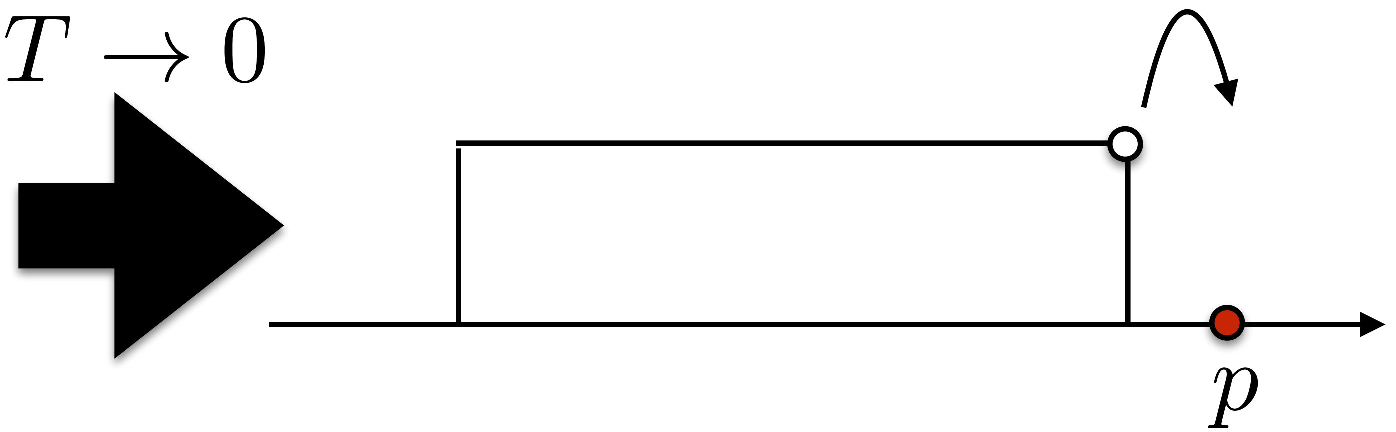
De Nardis and Panfil JSTAT (2015)

$$\langle \vartheta|\hat{\rho}|\vartheta, h \to p\rangle = \lim_{\mathrm{th}} L \frac{\langle \{\lambda_j\}^{\vartheta}|\hat{\rho}|\{\mu_j\}^{\vartheta}\rangle}{||\{\lambda_j\}^{\vartheta}||\ ||\{\mu_j\}^{\vartheta}||} \times e^{S_\mu - S_\lambda}$$

Comparison with results on the ground state

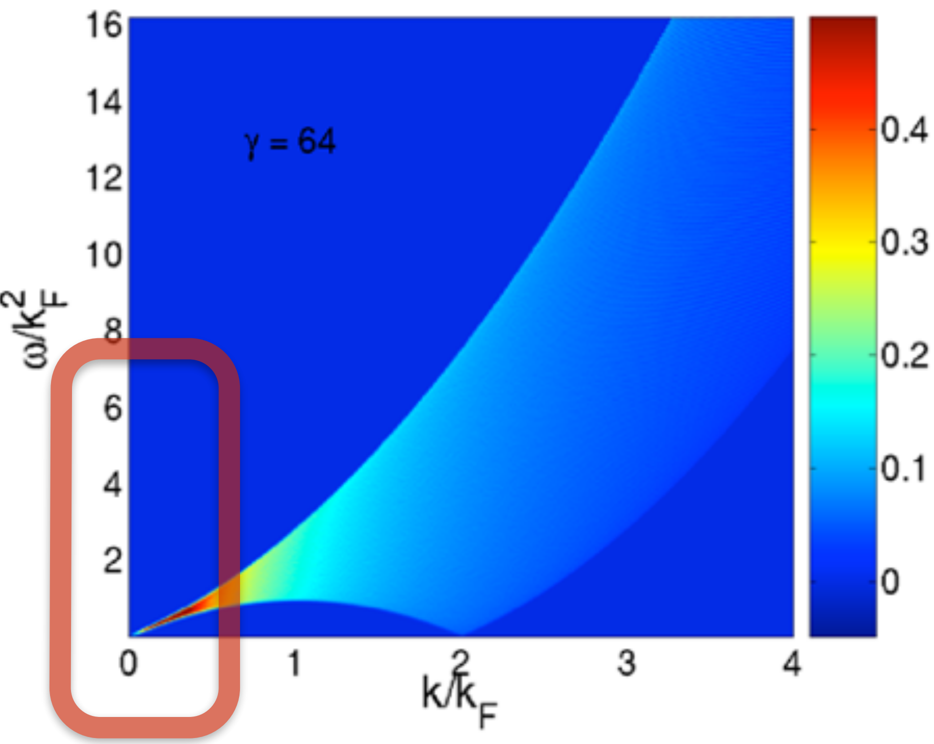
p

$T \to 0$



$$\lim_{T\to 0} S(k,\omega) = S_0(k,\omega)$$

De Nardis and Panfil in preparation

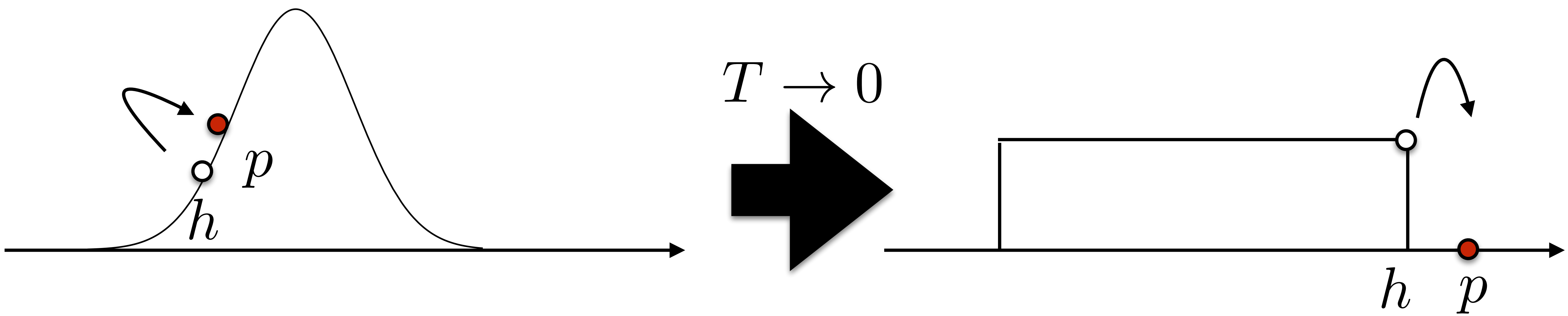


$$S_0(k,\omega) \sim A_{\pm}(k)(\omega^2 - \omega_{\pm}^2)^{\mp \frac{2k}{\sqrt{K}} L(q_F, q_F)}$$

Kitanine, Kozlowski, Maillet, Slavnov, Terras JSTATs (2009 - 2012)

Shashi et al PRB (2012)

Comparison with results on the ground state



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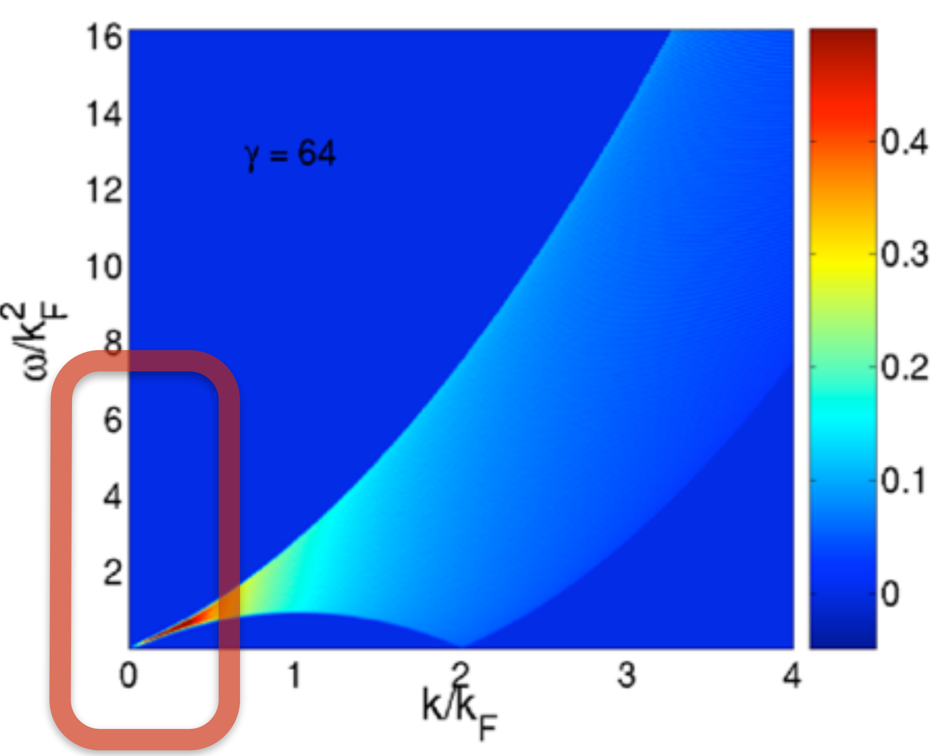
De Nardis and Panfil in preparation



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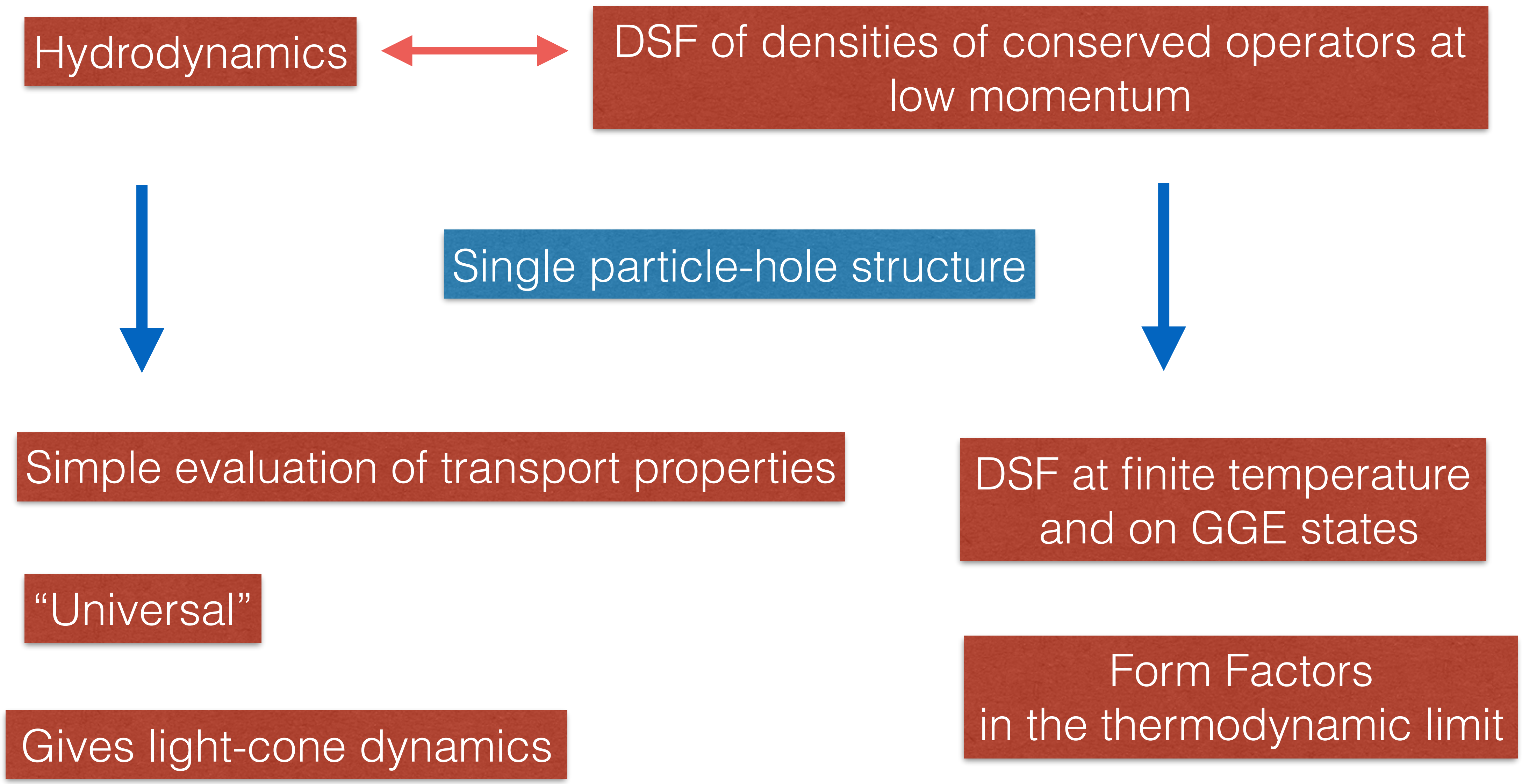
Kitanine, Kozlowski, Maillet, Slavnov, Terras JSTATs (2009 - 2012)

Shashi et al PRB (2012)

Conclusions



p

Conclusions

Hydrodynamics ↔ DSF of densities of conserved operators at low momentum

Single particle-hole structure

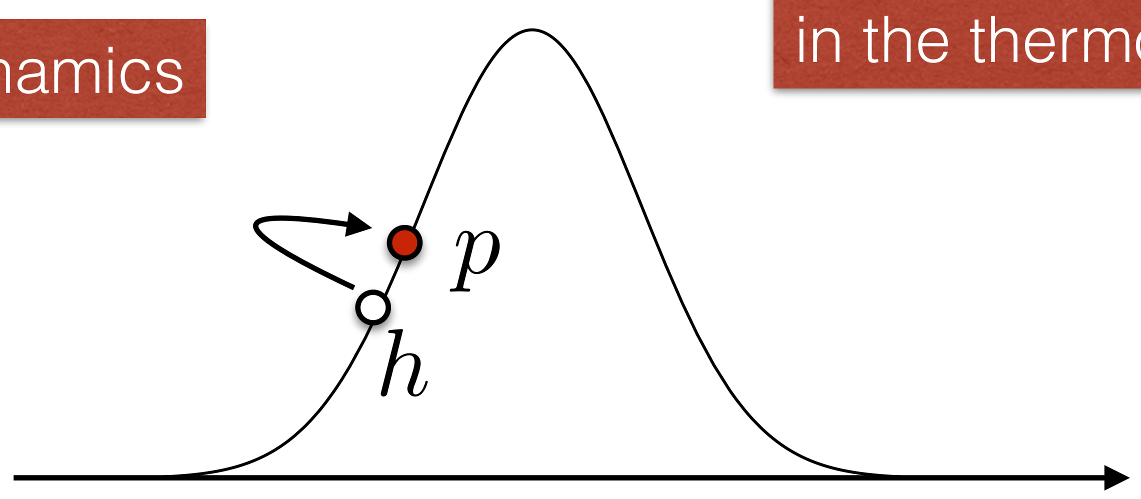
Simple evaluation of transport properties

"Universal"

Gives light-cone dynamics

DSF at finite temperature and on GGE states

Form Factors in the thermodynamic limit



Conclusions

Beyond linear Hydrodynamics ?

DSF of densities of conserved operators at higher momentum ?

Universal diffusive terms?

? More particle-hole or simply a new dressing?

Connections with field theory?

Proof of the current formula ?

Conclusions

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Proof of the current formula ?

$$\langle \vartheta | \hat{J}_s^{(n)} | \vartheta \rangle = \sum_a \int d\lambda \, \Big[\rho_a(\lambda) v_a(\lambda) \Big] q_{a,s}^{(n)}(\lambda)$$

Conclusions

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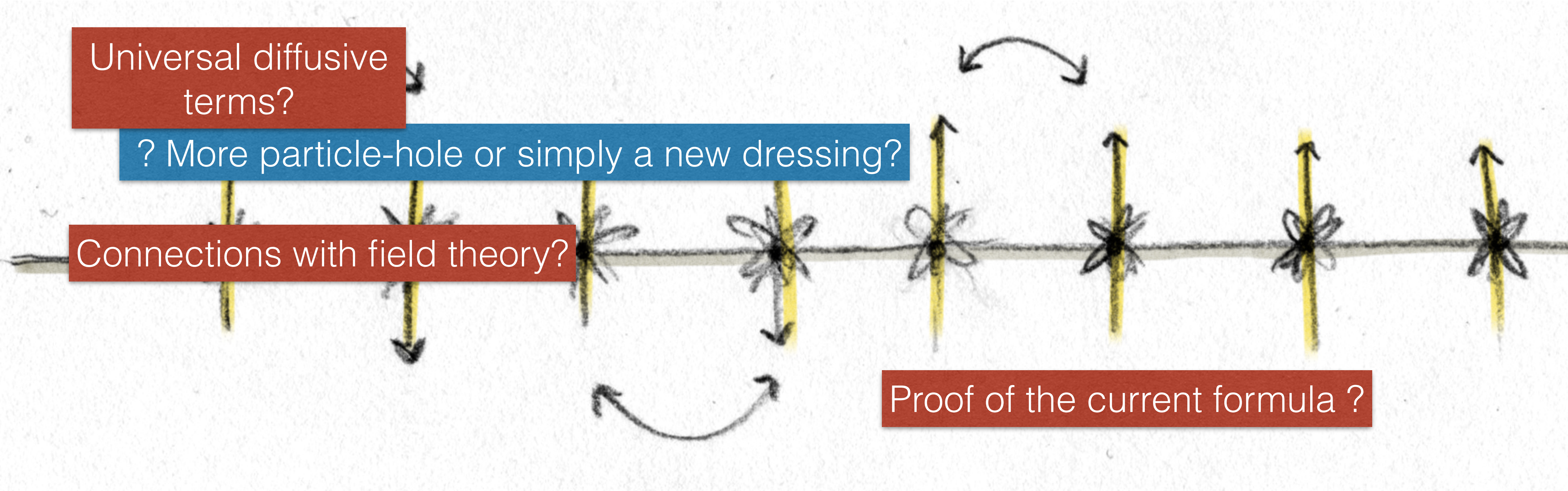

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Thank you.