Out-of-time-ordered correlators in locally interacting physics

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Out-of-time-ordered correlation functions (OTOC)

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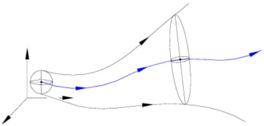
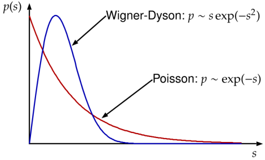
- Terms like: $\langle w(x,t)\, v(0)\, w(x,t)\, v(0) \rangle_\beta$

Quantum chaos?

- Classical chaos: sensitivity of the orbits to infinitesimal perturbations in initial conditions.
$$\lambda = \lim_{t\to\infty} \lim_{\delta Z_0 \to 0} \frac{1}{t} \ln \frac{|\delta Z(t)|}{|\delta Z_0|}$$
- In the quantum world only rigorously defined for systems that have a semi-classical limit and thus the notion of trajectories.



- Large part of the quantum chaos theory built on the random matrix theory - "quantum chaology". $\rightarrow$ So far, no "quantum Lyapunov exponent".



- The largest part of the theory limited to single particle systems $\rightarrow$ even less understood for quantum many-body systems.

Overview of the OTOC

- **Potentially the first genuinely quantum dynamical object to study (many-body) chaos.**
- Proposed in 2015 based on a 1969 work of Larkin and Ovchinnikov, soon became a hot topic.

SOVIET PHYSICS JETP VOLUME 28, NUMBER 6 JUNE, 1969

QUASICLASSICAL METHOD IN THE THEORY OF SUPERCONDUCTIVITY

A. I. LARKIN and Yu. N. OVCHINNIKOV

Institute of Theoretical Physics, USSR Academy of Sciences

Submitted June 6, 1968

Zh. Eksp. Teor. Fiz. 55, 2262–2272 (December, 1968)

time.

Let us consider how this commutator changes in the quasiclassical limit

$$\langle [p_z(t)p_z(0)]^2\rangle = h^2\left\langle\left(\frac{\partial \mathbf{p}_z(t)}{\partial z(0)}\right)^2\right\rangle, \tag{26}$$

where one can calculate the average in the right-hand side of formula (26) with respect to the classical trajectories. For the calculation we introduce the more

Thus, even for quasiclassical scattering of particles by impurities, the commutator of momentum operators at different moments of time increases exponentially with the time. For electrons in a metal, we apply the method of quasiclassical trajectories only in the pure case.

The problem considered above concerning the classical and quantum motion of an electron in a field

- For highly chaotic systems, the OTOC should grow exponentially:

$$C(x,t) \propto e^{\lambda_L (t - |x|/v_B)}$$

- The upper bound:

$$\lambda_L \leq \frac{2\pi}{\beta}$$

- Holography (AdS/CFT), large N gauge theories, Sachdev-Ye-Kitaev model:

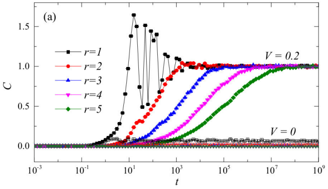
$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,l=1}^{N} J_{ij;kl} c_i^\dagger c_j^\dagger c_k c_l - \mu \sum_i c_i^\dagger c_i$$

- The OTOC are experimentally feasible.

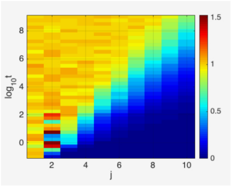
Systems with local interaction

- **Not a very exciting behavior of the OTOC** (Lieb-Robinson theorem):

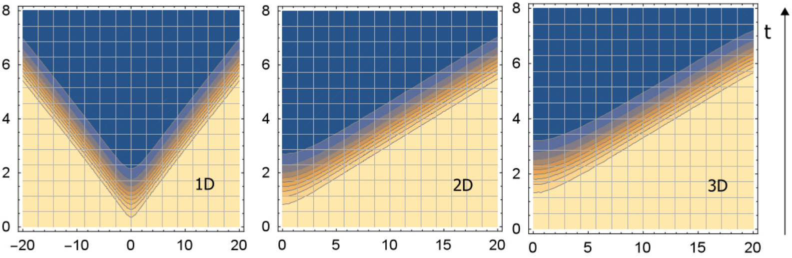
$$C\left(x,t\right)\leq 4\left\Vert v\right\Vert ^{2}\left\Vert w\right\Vert ^{2}e^{-\mu\max\{0,|x|-v_{LR}t\}}$$



Disordered nearest neighbor hopping Majorana fermions (MBL)



Random field XXX chain (MBL)



Interacting electron gas

dOTOC, Weak quantum chaos [Phys. Rev. B 96, 060301 (2017)]

- To be a well defined measure of chaos, the OTOC have to have a nontrivial $t \to \infty$ behavior. This can be achieved if the observables are unbounded.
- For example, one can take extensive observables

$$V \equiv \sum_{x \in \Lambda} v_x, \qquad W \equiv \sum_{x \in \Lambda} w_x,$$

with w_x, v_x local and study the density of the OTOC (dOTOC):

$$c^{(N)}(t) := -\frac{1}{N}\left(\langle [W(t), V(0)]^2 \rangle_\beta - \langle [W(t), V(0)] \rangle_\beta^2\right)$$

- Has the desired long-time properties.
- Proved that Lieb-Robinson theorem and exponential clustering property of thermal states imply that dOTOC can grow at most **polynomially in time in locally interacting lattice theories**:

$$c^{(N)}(t) \leq A t^{3d}$$

- In analogy to classical mixing systems with no butterfly effect: **weak quantum chaos**.

Kicked quantum Ising model

$$\begin{aligned} H(t) &= H_{\text{Ising}} + H_{\text{kick}} \sum_{n\in\mathbb{Z}} \delta(t-n) \\ &= \sum_j \left(J\sigma_j^x \sigma_{j+1}^x + h\left(\sigma_j^z \cos\theta + \sigma_j^x \sin\theta\right) \sum_{n\in\mathbb{Z}} \delta(t-n) \right) \end{aligned}$$

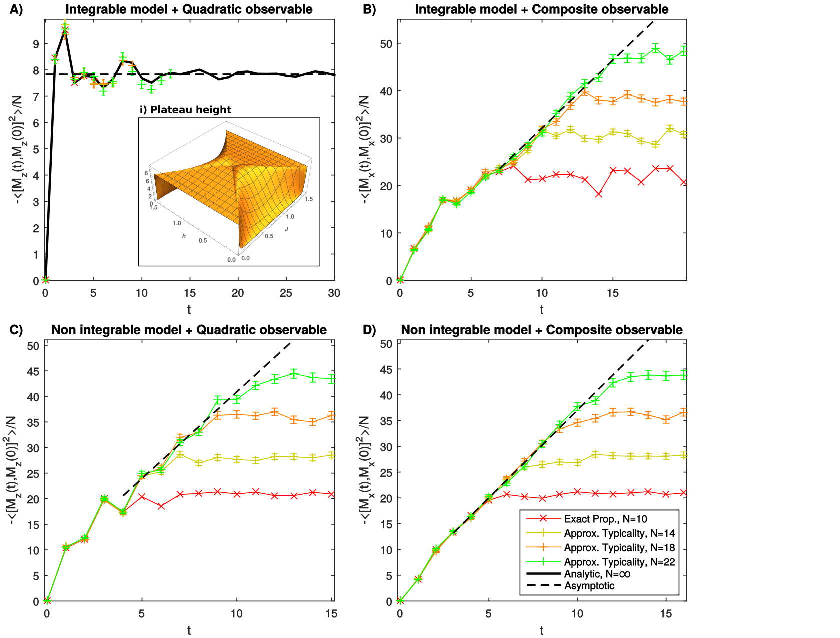
- A simple locally interacting model for which RMT shows that it is chaotic.
- A Floquet (time-periodical) system. These are themselves an active topic.
- For the transversal field ($\theta = 0$), the model is integrable and θ can serve as the parameter for integrability breaking.
- It is effectively time discrete - a quantum cellular automaton. It has a sharp light-cone spreading of information with velocity 1.

Methods

We are computing the dOTOC in three different ways:

- Numerically (for the general case - $\theta \in [0, \pi/2]$) with two different methods:
  1. Exact propagation for $N \sim 12$.
  2. Using typicality arguments: $\langle A \rangle \approx \frac{1}{|\{|\Psi_{\textbf{rand}}\rangle\}|} \sum_{\{|\Psi_{\textbf{rand}}\rangle\}} \langle \Psi_{\textbf{rand}} | A | \Psi_{\textbf{rand}} \rangle$ for $N \sim 22$.
- Analytically, using fermionization, for the integrable case $\theta = 0$ in the thermodynamic limit $N = \infty$.

Results



A) Integrable model + Quadratic observable
i) Plateau height
B) Integrable model + Composite observable
C) Non integrable model + Quadratic observable
D) Non integrable model + Composite observable
$-\langle [M_z(t), M_z(0)]^2 \rangle / N$
$-\langle [M_x(t), M_x(0)]^2 \rangle / N$
t
h
J
Exact Prop., N=10
Approx. Typicality, N=14
Approx. Typicality, N=18
Approx. Typicality, N=22
Analytic, N=∞
Asymptotic

- OTOC of local observables saturated to a plateau within a single kick of magnetic field.
- Two distinct behaviors of dOTOC: For a generic, nonintegrable case ($\theta \neq 0$) or if the observable is a sum of terms nonlocal in fermionic basis, OTOC grows (linearly) to infinity $\longrightarrow$ KI is weakly chaotic.
- For the integrable case and observables composed of operators quadratic in the fermionic basis, OTOC saturates to a plateau (din spite to the extensive observables). We prove that analytically.
- The plateau height is non-smooth at the Floquet phase transition line. Probably also the slope in the linearly growing cases.

Conclusions

- The OTOC as originally defined are probably not a good measure of chaos in locally interacting lattice theories.
- To get the required long-time behavior, one can take the density of the OTOC of extensive observables.
- In locally interacting theories the growth of such a quantity is always subexponential $\rightarrow$ weak quantum chaos.
- dOTOC can distinguish between different regimes of the KI model.
- Possible future research directions: dOTOC in the semiclassical regime, dependence on the range of interactions, connection to the transport properties,...