On diffusion in integrable systems

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Transport properties

- Microscopic origins of phenomenological laws ($\mathcal{D}$-diffusion constant, σ-conductivity, J-current, h-field, χ-static susceptibility)

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$$\mathcal{D} = \frac{\sigma}{\chi}$$

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    2. Diffusive (Heisenberg XXZ $\Delta > 1$, Hubbard)?
    3. Anomalous?

Lower bound on diffusion constant

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- No local charges with appropriate symmetry $\rightarrow$ absence of ideal transport
- The charges are absent for $\Delta \geq 1$

E. Ilievski, J. De Nardis, Phys. Rev. Lett. 119, 020602 (2017)

Connection of diffusion constant and Drude weight

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- Relative fraction of these states vanishes in TD limit
- Infinite conductivity from the states with measure zero, yields finite contribution to diffusion constant (D-Drude weight, the rate of the divergence of conductivity)
- Using LR theorem and clustering property of thermal state we get

$$\mathcal{D}(\beta) \geq \text{Const.} \times \frac{\partial^2}{\partial x^2} D(\beta, x)\Big|_{x=0},$$

Heisenber XXZ model

- Lower bounding the curvature of Drude weight:

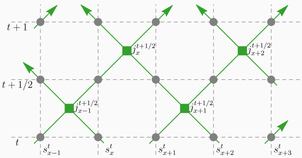
$$\begin{aligned} K^{x}_{s,s'} &= \lim_{n\to\infty} \frac{1}{n} \langle X_s(\lambda), X_{s'}(\mu)\rangle^{0,x}_n, \\ J^{x}_{s}(\lambda) &= \lim_{n\to\infty} \langle j, X_s(\lambda)\rangle^{0,x}_n, \\ J^{x}_{s'}(\mu) &= \sum_{s=1/2}^{\infty} \int d\lambda\ h^{x}_{s}(\lambda) K^{x}_{s,s'}(\lambda,\mu) \\ D(0,x) &= \sum_{s,s'=1/2}^{\infty} \int d\lambda \int d\mu\ K^{x}_{s,s'} h^{x}_{s}(\lambda) \bar{h}^{x}_{s'}(\mu). \end{aligned}$$

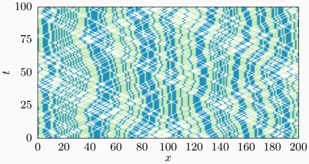
- Lower bound for Heisenberg from local IM ($\Delta = \cosh\gamma$):

$$\mathcal{D}(0) \geq \frac{\cosh(\gamma)}{3\ v_{\mathrm{LR}}} \left(e^{-\gamma} + \frac{2\sinh\gamma}{\sqrt{1+e^{2\gamma}+e^{4\gamma}}+2+e^{2\gamma}} \right)$$

M. Medenjak, K. Klobas, T. Prosen, Phys. Rev. Lett. 119, 110603 (2017)

- Classical cellular automaton
- Positive, negative charges (elastic scattering) and vacancies





Exact results for transport coefficient

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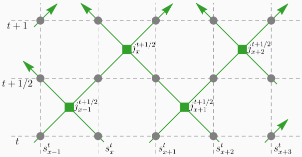

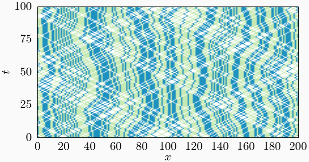

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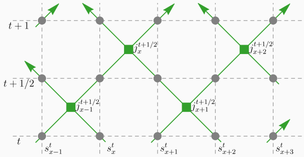

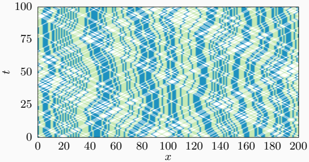

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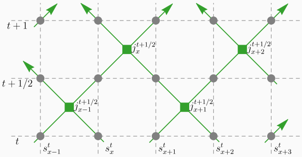

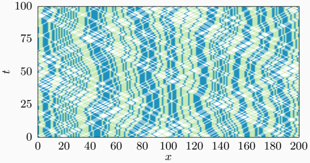

- Three regimes:
  1. Ballistic (Imbalance of charge $\mu \neq 0$)
  2. Diffusive (Density of particles $\rho \neq 1$)
  3. Isolating (Density of particles $\rho = 1$)

Exact results for transport coefficient

Central observations

- Conservation of total charge (imbalance subspace $\mathcal{A}^{(1)}$)
- P a linear projector $P : \mathcal{A}^{(1)} \to \mathcal{A}_J$
- $\langle J(1-P)a \rangle_p = 0$ for every $a \in \mathcal{A}^{(1)}$

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Imply

- The dynamics of correlation functions can then be restricted to $\mathcal{A}_J$, i.e. $\mathcal{U} = PSU^e P$

- Exact results on time dependent autocorrelation functions and inhomogeneous quench problem (choosing appropriate time-dependent basis)

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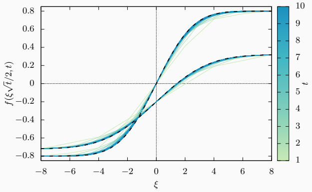
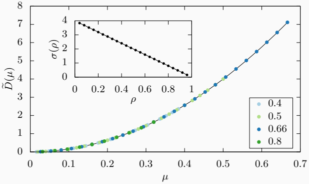


- Drude weight and diffusion constant



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- If we add tunneling (particles tunnel through each other with certain probability) $\rightarrow$ lower bound does not saturate the result.

Conclusion

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- Obtaining explicit time dependence for more complicated systems.