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MAX-PLANCK-GESELLSCHAFT

Long range correlations generated by phase separation. Exact results from field theory

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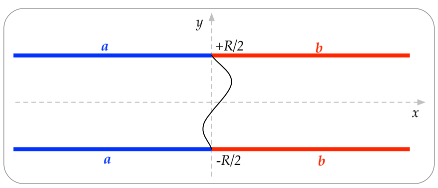
Aim & motivations

Fluctuating interfaces support long wavelength modes and correlation of the order parameter exhibit a long-range character in presence of phase separation. Although this implication has been pointed out since long time by the theory of inhomogeneous fluids [1], these correlations has never been derived within the underlying field theory of the scaling regime. Here we show the exact derivation for the planar case [2].

One-point function

We use symmetry breaking bc.s to induce phase separation in the bulk. We consider the near critical regime $R \gg \xi_{\text{bulk}} \gg a$; where a is the lattice spacing.



The interface is probed through the statistical average of the spin field $\sigma(x,y)$; an exact calculation reveals

$$\langle \sigma(x,y) \rangle_{ab} = \frac{\langle \sigma \rangle_a + \langle \sigma \rangle_b}{2} - \frac{\langle \sigma \rangle_a - \langle \sigma \rangle_b}{2} \text{erf}(\chi) + \frac{\mathcal{A}_{ab}}{m} P(x;y) + \dots$$

$\chi = \sqrt{\frac{2mR}{R^2-4y^2}}\, x$ and $P(x;y)$ is the (Gaussian) passage probability density [3].

- Abraham's exact solution for the Ising model [3] is recovered [4] with $\langle \sigma \rangle_+ = -\langle \sigma \rangle_-$;
- subleading effects ($\propto \mathcal{A}_{ab}(mR)^{-1/2}$) due to interfacial branching can be computed for integrable theories (as q-Potts) [3].

Two-point function

We consider the spin-spin pair correlation function. The exact calculation gives

$$\begin{aligned} \langle \sigma(x_1,y_1)\sigma(x_2,y_2) \rangle_{ab} &= \frac{(\langle \sigma \rangle_a - \langle \sigma \rangle_b)^2}{4} \mathcal{G}(\eta_1,\varepsilon_1;\eta_2,\varepsilon_2) + \\ + \frac{(\langle \sigma \rangle_a + \langle \sigma \rangle_b)^2}{4} &+ \frac{\langle \sigma \rangle_b^2 - \langle \sigma \rangle_a^2}{4} \left[\text{erf}(\chi_1) + \text{erf}(\chi_2) \right], \end{aligned}$$

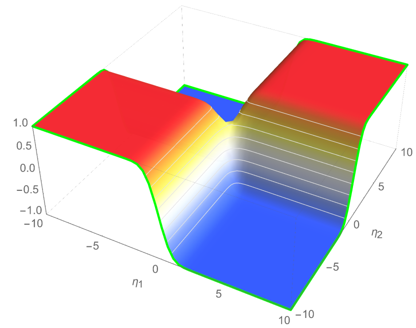
where $\eta_j = \sqrt{2m/R}\, x_j$, $\chi_j = \eta_j/\kappa_j$, $\kappa_j = \sqrt{1-\varepsilon_j^2}$, $\varepsilon_j = 2y_j/R$, $\mathcal{G}$ is the (smooth) function

$$\mathcal{G}(\eta_1,\varepsilon_1;\eta_2,\varepsilon_2) = \text{sign}(\eta_1\eta_2) - 4\sum_{j=1}^{2} \mathcal{T}(\sqrt{2}\chi_j, q_j),$$

$\mathcal{T}$ is Owen's T function and

$$q_{1,2} = \sqrt{\frac{(1-\varepsilon_1)(1+\varepsilon_2)}{2(\varepsilon_1-\varepsilon_2)}} \left(\frac{\eta_{2,1}}{\eta_{1,2}} \frac{1 \pm \varepsilon_{1,2}}{1 \pm \varepsilon_{2,1}} - 1 \right)$$

The scaling function $\mathcal{G}(\eta_1,\varepsilon;\eta_2,-\varepsilon)$ for $\varepsilon = 0.3$ is plotted:



Long-range correlations in the interfacial region are neatly seen from

$$\langle \sigma(x,y)\sigma(x,-y) \rangle_{-+} = \langle \sigma \rangle^2 \left(1 - \frac{4}{\pi} \sqrt{\frac{2y}{R}} \mathrm{e}^{-2mx^2/R} \right) + \dots$$

for the Ising model with $\xi \ll y \ll R/2$.

Interface structure factor

Interfacial fluctuations can be characterized in momentum space through the interface structure factor $\mathcal{S}(q)$. For a system defined on a finite strip

$$\mathcal{S}(q) = \frac{1}{2(\Delta\langle\sigma\rangle)^2} \int_{-R/2}^{R/2} \mathrm{d}y\, \mathrm{e}^{-iqy} \int_{\mathbb{R}^2} \mathrm{d}x_1 \mathrm{d}x_2 \langle \sigma(x_1,y)\sigma(x_2,-y) \rangle_{ab}^{\text{CP}}$$

$\Delta\langle\sigma\rangle = \langle\sigma\rangle_a - \langle\sigma\rangle_b$, CP = connected part. The above contains only interfacial degrees of freedom. The exact result is

$$\begin{aligned} \mathcal{S}(q) &= \frac{1}{mq^2} + \frac{\mathcal{A}_{ab}^2 \sin Q}{m^2 q} + \frac{Y(Q,m,q)}{mR} + \mathcal{O}(R^{-2}) \\ Y(Q,m,q) &= 4\alpha_2^2 \frac{\sin Q}{m^2 q} + 4\alpha_2 \frac{\cos Q}{mq^2} - \frac{2\sin Q}{q^3}, \quad Q = qR/2, \end{aligned}$$

α_2 is a boundary parameter. The factor $\mathcal{A}_{ab}$ vanishes for the Ising model but it differs from zero for the q-Potts model with $q = 3, 4$.

Conclusions

- ✓ $\mathcal{S}(q)$ is derived from first principles, for a broad category of universality classes, including its finite-size corrections for finite R and specific features of the underlying universality class. The result is exact and holds for $R \gg \xi_{\text{bulk}}$;
- ✓ Capillary Wave Theory is reproduced at the leading order;
- ✓ For $R = \infty$: the (infinite) fluctuations lead to the averaging of exponential correlations over two bulk phases and the subleading corrections localize towards $q = 0$.

References

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